Name

10-7 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 673–677 for objectives.

SKILLS Objectives C and D

In 1–4, find the dot product of the two vectors.

1. $\langle -1, 4\rangle$ and $\langle 7, -2\rangle$ **−15**
2. $\langle 3, 4\rangle$ and $\langle 9, 0\rangle$ **27**
3. $\langle 0.4, 0.8\rangle$ and $\langle 4.5, 1\rangle$ **2.6**
4. $\left\langle \frac{1}{2}, -9\right\rangle$ and $\left\langle 6, -\frac{2}{3}\right\rangle$ **9**

In 5–7, find the measure of the angle between the vectors given in the indicated question.

5. Question 1 $\cos^{-1}\left(\frac{-15}{\sqrt{17}\sqrt{53}}\right) \approx 120°$
6. Question 2 $\cos^{-1}\left(\frac{27}{45}\right) \approx 53.1°$
7. Question 3 $\cos^{-1}\left(\frac{9}{\sqrt{0.8}\sqrt{21.25}}\right) \approx 50.9°$

PROPERTIES Objectives E and F

8. Let $\vec{u}$ and $\vec{v}$ be any two-dimensional vectors and k be any nonzero real number. Prove that $k\vec{u} \cdot k\vec{v} = k^2(\vec{u} \cdot \vec{v})$.

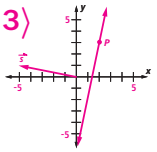
Let $\vec{u} = \langle u_1, u_2\rangle$ and $\vec{v} = \langle v_1, v_2\rangle$. By the definition of scalar multiplication, $k\vec{u} = \langle ku_1, ku_2\rangle$ and $k\vec{v} = \langle kv_1, kv_2\rangle$. By the definition of dot product, $k\vec{u} \cdot k\vec{v} = ku_1kv_1 + ku_2kv_2 = k^2u_1v_1 + k^2u_2v_2 = k^2(u_1v_1 + u_2v_2) = k^2(\vec{u} \cdot \vec{v})$.

9. Tell whether each pair of vectors are orthogonal.
   a. $\langle -3, -7\rangle$ and $\langle 6, -2\rangle$ **no**
   b. $\langle 12, 3\rangle$ and $\langle -1, 4\rangle$ **yes**
10. Let $\vec{w} = \langle 4, 9\rangle$ and $\vec{s} = \langle -2, y\rangle$. Find y so that $\vec{w}$ and $\vec{s}$ are orthogonal. $y = \frac{8}{9}$
11. Find all two-dimensional vectors $\vec{v}$ that are orthogonal to $\langle 6, 4\rangle$ and have magnitude 5.

$\vec{v} = \left\langle \frac{10}{\sqrt{13}}, -\frac{15}{\sqrt{13}}\right\rangle$ or $\vec{v} = \left\langle -\frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}}\right\rangle$

REPRESENTATIONS Objective N

12. Let $P = (2, 3)$ and $\vec{s} = \langle -5, 1\rangle$.
    a. Find a vector equation for the line through P that is orthogonal to $\vec{s}$. $\langle x - 2, y - 3\rangle = t\langle 1, 5\rangle$
    b. Find parametric equations for the same line. $\begin{cases} x = 2 + t \\ y = 3 + 5t \end{cases}$
    c. Sketch a graph of P, $\vec{s}$, and the line on the axes at the right.



Name

11-1 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 733–737 for objectives.

REPRESENTATIONS Objective I

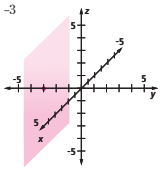
In 1–3, write an equation for the plane(s) described.

Answers vary. Sample:

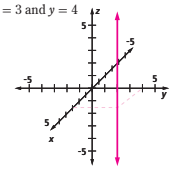
1. the yz-plane $x = 0$
2. a plane parallel to the z-axis $2x + y = 0$
3. all planes parallel to and 6 units from the xz-plane $y = 6; y = -6$

In 4–7, sketch a graph of all the points (x, y, z) that satisfy the equation or equations.

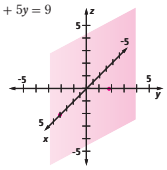
4. $y = -3$



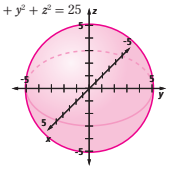
5. $x = 3$ and $y = 4$



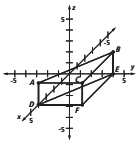
6. $3x + 5y = 9$



7. $x^2 + y^2 + z^2 = 25$



8. Verify that the equation $x^2 + y^2 + z^2 + 3x - 2y + 6z = 18$ describes a sphere.
   a. Complete the square: $x^2 + 3x + y^2 - 2y + z^2 + 6z = 18$
   $x^2 + 3x + 2.25 + y^2 - 2y + 1 + z^2 + 6z + 9 = 18 + 2.25 + 1 + 9$
   $(x + 1.5)^2 + (y - 1)^2 + (z + 3)^2 = 30.25$; Thus, this is an equation for a sphere by the Equation for a Sphere Theorem.
   b. Give the center and radius of the sphere. center: $(-1.5, 1, -3)$; radius: 5.5
9. Refer to the graph at the right.
   a. Find the coordinates of point A. $(4, 0, 2)$
   b. Find an equation for the plane containing points A, B, and C. $z = 2$
   c. Find an equation for the plane containing points A, B, D, and E. $x + y = 4$



Name

11-2 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 733–737 for objectives.

SKILLS Objectives A and B

1. Let $\vec{u}$ and $\vec{v}$ be any two 3-dimensional vectors. Tell whether each of the following is a vector or a real number.
   a. $-\vec{u}$ **vector**
   b. $\vec{u} \cdot \vec{v}$ **real number**
   c. $\|\vec{v}\|$ **real number**

In 2–7, let $\vec{u} = \langle 1, -2, 4\rangle$ and $\vec{v} = \langle 6, 1, 3\rangle$ and compute.

2. $\vec{u} + \vec{v}$ $\langle 7, -1, 7\rangle$
3. $\vec{u} - \vec{v}$ $\langle -5, -3, 1\rangle$
4. $3\vec{v}$ $\langle 18, 3, 9\rangle$
5. $2\vec{u} - 3\vec{v}$ $\langle -16, -7, -1\rangle$
6. $\|\vec{v}\|$ $\sqrt{46}$
7. $\vec{u} \cdot \vec{v}$ **16**

In 8 and 9, find the exact and approximate measure of the angle between the two vectors.

8. $\langle 3, 2, -1\rangle$ and $\langle 0.5, 0, 6\rangle$ $\cos^{-1}\left(\frac{-4.5}{\sqrt{14}\sqrt{36.25}}\right) \approx 101.52°$
9. $\langle -2, 9, 4\rangle$ and $\langle -1, 4, -3\rangle$ $\cos^{-1}\left(\frac{26}{\sqrt{101}\sqrt{26}}\right) \approx 59.51°$

PROPERTIES Objective E

10. Prove: For any 3-dimensional vectors $\vec{u}$ and $\vec{v}$ and any real number k, $k\vec{u} \cdot \vec{v} = \vec{u} \cdot k\vec{v}$.

Let $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$.

Then $k\vec{u} = \langle ku_1, ku_2, ku_3\rangle$ and $k\vec{v} = \langle kv_1, kv_2, kv_3\rangle$.

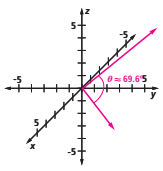
Thus, $k\vec{u} \cdot \vec{v} = ku_1v_1 + ku_2v_2 + ku_3v_3 = u_1kv_1 + u_2kv_2 + u_3kv_3 = \vec{u} \cdot k\vec{v}$.

USES Objective G

11. Joan pushed a grocery cart down a 150-ft grocery aisle. If she applied 28 lb of force to the cart handle at an angle of 15° with the horizontal, how much work did she do? $4200 \cos 15° \approx 4057$ ft-lb
12. If Eddie is pulls his little sister on a sled 400 ft across a field by pulling with 34 lb of force at an angle of 60° with the horizontal, how much work will he do? $13{,}600 \cos 60° = 6800$ ft-lb

REPRESENTATIONS Objective H

13. On the axes at the right, draw the vectors $\langle -2, 4, 3\rangle$ and $\langle 4, 5, 0\rangle$ in standard position and label the angle between them.



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11-3 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 733–737 for objectives.

SKILLS Objective A

In 1 and 2, find the cross product of the two given vectors.

1. $\left\langle \frac{1}{2}, 2, \frac{2}{3}\right\rangle$ and $\langle -2, 0, 1\rangle$ $\left\langle 2, -\frac{11}{6}, 4\right\rangle$
2. $\langle -1, 3, -2\rangle$ and $\langle 6, 2, 4\rangle$ $\langle 16, -8, -20\rangle$
8. Find a vector orthogonal to both $\langle 0.5, 2, 3\rangle$ and $\langle -3, 7, 0.1\rangle$. Answers vary. Sample: $\langle -20.8, -9.05, 9.5\rangle$

PROPERTIES Objectives E and F

4. Prove algebraically that for any 3-dimensional vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$.

Let $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$.

Then $\vec{u} \times \vec{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_2v_1 - u_1v_2\rangle =$
$\langle -(u_3v_2 - u_2v_3), -(u_1v_3 - u_3v_1), -(u_2v_1 - u_1v_2)\rangle =$
$-\langle u_3v_2 - u_2v_3, u_1v_3 - u_3v_1, u_1v_2 - u_2v_1\rangle = -(\vec{v} \times \vec{u})$.

In 5 and 6, determine if the two given vectors are orthogonal.

5. $\langle 1, -1, 1\rangle$ and $\langle 4, 8, 4\rangle$ **yes**
6. $\langle 0.3, -1, 4\rangle$ and $\langle 0, -2, -2\rangle$ **no**

In 7 and 8, find t so that the two vectors are orthogonal.

7. $\left\langle 7, \frac{1}{2}, t\right\rangle$ and $\langle -3, 20, 1\rangle$ $t = 11$
8. $\langle 9, -3, 7\rangle$ and $\langle 2, t, -5\rangle$ $t = -\frac{17}{3}$

USES Objective G

9. The lid of a new jar of pickles is sealed so that it holds 12 ft-lb of torque. Alyson grips the lid using a tool that is 6 inches long and applies force perpendicular to the end of the tool.
   a. How much force does she need to apply to open the jar? 24 lb
   b. Assume the center of the jar lid is the origin of a coordinate system and the lid is in the xy-plane. If the gripping tool starts along the positive x-axis and Alyson applies the force in the direction of the positive y-axis, find the torque vector. $\langle 0, 0, 12\rangle$

REPRESENTATIONS Objective H

10. On the axes at the right, sketch the vectors in Question 1 and their cross product.
11. Find the area of the parallelogram determined by the vectors in Question 2. $\sqrt{720} = 12\sqrt{5}$