Name

10-3 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 673–677 for objectives.

USES Objective I

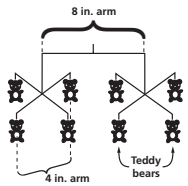
1. A large clock is used for part of a backdrop for a play. The circular clock is 4 feet tall and mounted so that the bottom of it is 5 feet from the stage floor. To simulate quicker passage of time, the clock is engineered so that the minute hand makes a full revolution every 8 minutes. The minute hand stretches from the center to the edge of the clock.

   a. If the origin is the point on the stage floor directly below the center of the clock, find parametric equations to model the position of the tip of the minute hand t minutes after leaving the 12.

   $\begin{cases} x = 2\sin\left(\frac{\pi}{4}t\right) \\ y = 2\cos\left(\frac{\pi}{4}t\right) + 7 \end{cases}$

   b. Use the equations to find the position of the tip of the minute hand after 2 hours and 14 minutes, the running time of the play. about 7 feet above the stage floor and 2 feet to the left of the center of the clock

2. A battery-powered baby mobile has an 8-inch rotating arm. At each end of the main arm, there are two perpendicular 4-inch arms that rotate 3 times for every rotation of the main arm. At the end of each 4-inch arm, there is a teddy bear.



   a. Find parametric equations for the path of one of the ends of the main arm relative to the center of the main arm. Use the magnitude θ of rotation as the parameter.

   $\begin{cases} x = 4\cos\theta \\ y = 4\sin\theta \end{cases}$

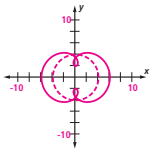
   b. Find parametric equations in θ for the path of one of the teddy bears relative to the end of the main arm.

   $\begin{cases} x = 2\cos(3\theta) \\ y = 2\sin(3\theta) \end{cases}$

   c. Use your answers to Parts a and b to find parametric equations for the path of a teddy bear relative to the center of the main arm.

   $\begin{cases} x = 4\cos\theta + 2\cos(3\theta) \\ y = 4\sin\theta + 2\sin(3\theta) \end{cases}$

   d. Graph your answers to Parts a and c using the axes at the right. Use a dashed line to represent the path of the end of the main arm.



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10-4 Lesson Master

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SKILLS Objective B

In 1–3, find the magnitude and direction of the given vector.

1. $\langle -2, 7\rangle$ $\sqrt{53} \approx 7.28$; $180° - \tan^{-1}\left(\frac{7}{2}\right) \approx 105.9°$
2. the vector from $(1, 4)$ to $(3, 19)$ $\sqrt{229} \approx 15.13$; $\tan^{-1}\left(\frac{15}{2}\right) \approx 82.4°$
3. the vector in standard position with its tip at $(-5, -12)$ 13; $180 + \tan^{-1}\left(\frac{12}{5}\right) \approx 247.4°$
4. a. For a certain vector, its x-component is $\frac{1}{4}$ of its y-component. Find all possible directions for this vector. $\tan^{-1}4 \approx 75.96°$ or $225.96°$

   b. Find the magnitude of this vector in terms of its x-component. $\sqrt{17x^2}$

USES Objective G

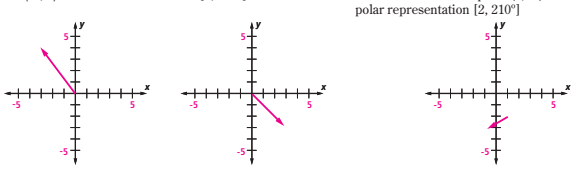
In 5 and 6, a description of a vector is given.
a. Give a polar representation of the vector. b. Give a component representation of the vector.
c. Interpret the components of the vector in context.

5. A cheetah is running 10 degrees south of west at a rate of 65 miles per hour.
   a. $[65, 190°]$ b. $\langle 65\cos 190°, 65\sin 190°\rangle \approx \langle -64.0, -11.3\rangle$
   c. The cheetah is traveling west at about 64 mph and south at 11.3 mph.
6. A 24-foot guy wire supporting a tent pole makes a 72° angle with the ground.
   a. $[24, 72°]$ b. $\langle 24\cos 72°, 24\sin 72°\rangle \approx \langle 7.4, 22.8\rangle$
   c. The pole is about 7.4 feet from where the wire meets the ground, and the wire is attached to a point on the pole about 23 feet from the ground.

REPRESENTATIONS Objectives L

In 7–9, sketch the vector whose description is given.

7. $\langle -3, 4\rangle$
8. $[4, 315°]$
9. the vector with initial point $(1, -2)$ and polar representation $[2, 210°]$



Name

10-5 Lesson Master

Questions on SPUR Objectives
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SKILLS Objective C

In 1–3, let $\vec{u} = \langle 16, 2\rangle$ and $\vec{v} = \langle -4, 7\rangle$. Find each vector in component form.

1. $\vec{u} + \vec{v}$ $\langle 12, 9\rangle$
2. $\vec{u} - \vec{v}$ $\langle 20, -5\rangle$
3. $-\vec{u}$ $\langle -16, -2\rangle$

In 4–6, let $\vec{u} = [5, 30°]$ and $\vec{v} = [2, 135°]$. Find each vector in polar form.

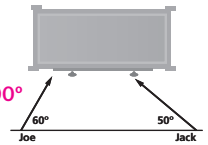
4. $\vec{u} + \vec{v} \approx [4.88, 53.3°]$
5. $\vec{u} - \vec{v} \approx [5.84, 349.3°]$
6. $-\vec{u}$ $[5, 210°]$

PROPERTIES Objective E

7. Prove that vector addition is commutative; that is, prove that for all vectors $\vec{u}$ and $\vec{v}$, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.
Let $\vec{u} = \langle u_1, u_2\rangle$ and $\vec{v} = \langle v_1, v_2\rangle$. Then by the definition of vector addition, $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2\rangle$ and $\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2\rangle$. Thus by commutativity of addition of real numbers, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.

USES Objective H

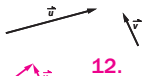
8. The instrument panel on an airplane indicates that the plane is flying at 270 mph in a direction 30° west of south. There is a wind blowing from the northeast at 30 mph.
   a. Find vectors in polar form to represent the direction and speed of the plane (relative to still air) and the wind. plane: $[270, 240°]$; wind: $[30, 225°]$
   b. Add the vectors in Part a, and explain what the resultant vector means for the plane. $\approx [299, 238.5°]$; The plane is actually flying at about 299 mph in a direction 31.5° west of south.
9. Twin brothers Jack and Joe share a room with their older brother Jim. They are trying to push Jim's dresser back onto his side of the room. Jack applies 32 lb of force and Joe applies 26 lb of force in the directions shown.



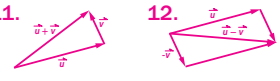
   a. Find the magnitude and direction of the force the twins apply together. about 48.2 pounds; $\approx 100°$
   b. Find a vector for the force Jim must apply to keep the dresser stationary. $[48.2, 280°]$

REPRESENTATIONS Objective M

In 10–12, use the vectors $\vec{u}$ and $\vec{v}$ at the right.



10. Sketch $-\vec{u}$.
11. Sketch a diagram showing how to find $\vec{u} + \vec{v}$.
12. Sketch a diagram showing how to find $\vec{u} - \vec{v}$.



Name

10-6 Lesson Master

Questions on SPUR Objectives
See Student Pages pages 673–677 for objectives.

SKILLS Objective C

1. Let $\vec{u} = \langle 4, 7\rangle$ and $\vec{v} = \langle -2, 0.5\rangle$. Find each vector in component form.
   a. $3\vec{u}$ $\langle 12, 21\rangle$ b. $-4\vec{v}$ $\langle 8, -2\rangle$
   c. $-\vec{u} + 2\vec{v}$ $\langle -8, -6\rangle$
2. Let $\vec{w} = [0.8, 12°]$ and $\vec{z} = [12, 47°]$. Find each vector in polar form.
   a. $-6\vec{w}$ $[4.8, 192°]$ b. $\frac{1}{3}\vec{z}$ $[4, 47°]$

PROPERTIES Objectives E and F

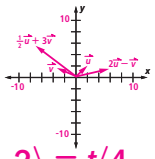
3. Prove that for all vectors $\vec{u}$ and $\vec{v}$ and all real numbers k, $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$.
Let $\vec{u} = \langle u_1, u_2\rangle$ and $\vec{v} = \langle v_1, v_2\rangle$. Then by the definition of scalar multiplication, $k(\vec{u} + \vec{v}) = k\langle u_1 + v_1, u_2 + v_2\rangle = \langle k(u_1 + v_1), k(u_2 + v_2)\rangle = \langle ku_1 + kv_1, ku_2 + kv_2\rangle = \langle ku_1, ku_2\rangle + \langle kv_1, kv_2\rangle = k\vec{u} + k\vec{v}$.

In 4–6, tell whether the two given vectors are parallel and justify your answer.

4. $\langle 1, -2\rangle$ and $\langle -6, 12\rangle$ Yes, $\langle -6, 12\rangle = -6\langle 1, -2\rangle$.
5. $[13, 17°]$ and $[5, 163°]$ No, $163 \neq 17 \bmod 180$.

REPRESENTATIONS Objectives M and N

6. Use the grid at the right and label your sketches.



   a. Sketch $\vec{u} = [2, 45°]$ and $\vec{v} = [3, 150°]$.
   b. Sketch $2\vec{u} - \vec{v}$.
   c. Sketch $\frac{1}{2}\vec{u} + 3\vec{v}$.
7. Let $\vec{v} = \langle 4, -3\rangle$ and $P = (-1, 2)$.
   a. Find a vector equation for the line through P parallel to $\vec{v}$. $\langle x + 1, y - 2\rangle = t\langle 4, -3\rangle$
   b. Find parametric equations for the line in Part a. $\begin{cases} x = -1 + 4t \\ y = 2 - 3t \end{cases}$
   c. Find an equation for the line in Parts a and b in point-slope form. $y - 2 = -\frac{3}{4}(x + 1)$
8. Find a vector equation for the line with parametric equations $\begin{cases} x = 2 + 0.5t \\ y = 8 - 2t \end{cases}$. $\langle x - 2, y - 8\rangle = t\langle 0.5, -2\rangle$