Let $z = \pm 5 - 5\sqrt{3}i$.

1. Write z in rectangular form.

2. Write z in polar form.

Let $z = 3\sqrt{2} - 3\sqrt{2}i$.

3. Write z in rectangular form.

4. Write z in polar form.

Let $z = 3 \pm 5i$ and $w = 11 + 2i$.

5. Write $z + w$ in $a + bi$ form.

6. Write $\frac{w}{z}$ in $a + bi$ form.

7. Let $z = \pm 6 + 7i$ and $w = 3 \pm 4i$.
   Write $\frac{z}{w}$ in $a + bi$ form.

8. Let $z = \pm 5 - 5\sqrt{3}i$.
   Write z^5 in polar form.

9. Find all solutions and their multiplicities for $p(x) = \left(x^2 + 4x + 4\right)\left(x^4 - 16\right)$.

10. Find all solutions and their multiplicities for $p(x) = \left(x^4 - x^2 - 12\right)\left(x^4 - 4x^2\right)$.

11. Prove that the product of a complex number and its conjugate is a real number.

12. Let $z = 3 + 2i$ and $w = 5 - 7i$. Verify that $\overline{z - w} = \overline{z} - \overline{w}$.

13. Without doing any computation, explain why 5, $2i$, and $2 \pm i$ cannot all be zeros of the polynomial $p(x) = x^3 - 5x^2 + 4x - 20$.

14. If $\pm 3 \pm 4i$ is a zero of polynomial function $q(x)$ with real coefficients, then name another zero of the function.

15. Consider the transformation Q defined for all complex numbers z by $Q(z) = 2z \pm 3 + 5i$. Is Q an isometry? If it is, prove that it is. If it is not, explain why it is not.

16. Consider the transformation P defined for all complex numbers z by $P(z) = \overline{z} + 5 - 2i$. Is P an isometry? If it is, prove that it is. If it is not, explain why it is not.

17. Calculate the first six values in the orbit of $x_0 = 0.352$ under $f(x) = \dfrac{1}{x+1}$.

18. Calculate the first six values in the orbit of $x_0 = 0.8$ under $f(x) = (1 - x)^2$.

19. The first seven values in the orbit of $x_0 = 0.286$ under $f(x) = \dfrac{1}{x+1}$ are 0.778, 0.562, 0.640, 0.610, 0.621, 0.617, and 0.618. Does this orbit seem to converge to some value? If so, what value?

20. The first six values in the orbit of $x_0 = 0.7$ under $f(x) = (1 - x)^2$ are 0.09, 0.8281, 0.0295, 0.9418, 0.0034, 0.9932. Does this orbit seem to converge to some value? If so, what value?

21. A bag of chicken is taken from the freezer at a temperature of 0°F. It is put in the sink of a 72°F kitchen to thaw. Suppose the difference between the two temperatures decreases 15% every 30 minutes. Describe a discrete dynamical system that models this situation.

22. A bag of chicken is taken from the freezer and is put in the sink to thaw. Suppose a dynamical system that describes this situation is $f(x) = x + 0.20(74 - x)$ with $x_0 = 0$. Calculate the first six terms of the orbit of the system. Round each term to the nearest hundredth.

23. A bag of chicken is taken from the freezer and is put in the sink to thaw. Suppose the difference between the temperature of the chicken and the temperature of the air decreases 18% every 30 minutes. The first six terms of the orbit of the discrete dynamical system that models this situation are 0, 12.6, 22.93, 31.40, 38.35, and 44.05. According to this model, about how long will it take the chicken to reach the freezing point of 32°F?

24. A mother fills a wading pool for her toddler with water from an outdoor faucet with no temperature control and finds the temperature to be 71°F. This feels too cold, so she decides to let it sit and warm up, since the weather is a balmy 84°F. Suppose the difference between the two temperatures decreases 10% every 20 minutes. Describe a discrete dynamical system that models this situation.

25. A mother fills a wading pool for her toddler with water from an outdoor faucet and finds the temperature to be too cold, so she decides to let it sit and warm up. Suppose a discrete dynamical system that models this situation is $f(x) = x + 0.12(86 - x)$ with $x_0 = 72$. Calculate the first six terms in the orbit of the system. Round each term to the nearest hundredth.

26. A mother fills a wading pool for her toddler with water from an outdoor faucet and finds the temperature to be too cold, so she decides to let it sit and warm up. Suppose the difference between the water temperature and the air temperature decreases 15% every 20 minutes. The first six terms in the orbit of the discrete dynamical system that models this situation are 74, 75.8, 77.33, 78.63, 79.74, and 80.68. If the mother decides to let her toddler swim after an hour, what will the temperature of the water be according to this model?

27. Illustrate the multiplication of $z = 3(\cos 35° + i\sin 35°)$ by $w = 2(\cos 106° + i\sin 106°)$ with a diagram that verifies the Geometric Multiplication Theorem.

28. Let $z = \left[\frac{4}{3}, \frac{\pi}{4}\right]$. Calculate z^1, z^2, z^3, and z^4.

Suppose $z = \left[\frac{4\sqrt{2}}{3}, \frac{\pi}{4}\right]$ so that $z^1 = \left[\frac{4\sqrt{2}}{3}, \frac{\pi}{4}\right]$, $z^2 = \left[\frac{32}{9}, \frac{\pi}{2}\right]$, $z^3 = \left[\frac{128\sqrt{2}}{27}, \frac{3\pi}{4}\right]$, and $z^4 = \left[\frac{1024}{81}, \pi\right]$.

29. Graph these powers in the complex plane.

30. Will the graphs of these powers get closer to or farther away from 0 as the exponent increases?

31. Find the fifth roots of 243 and plot them in the complex plane.

32. Consider the transformation Q defined for all complex numbers z by $Q(z) = 2z \pm 3 + 5i$. Q can be written as $Q = T_{a+bi} \circ S_k$. Find a, b, and k.