

**LESSON  
MASTER****4-1****Questions on SPUR Objectives**

See pages 303–307 for objectives.

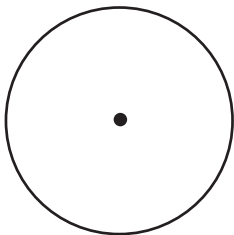
**Skills** Objective A**In 1 and 2, the measure of a rotation is given.****a. Convert the measure to revolutions.      b. On the circle draw a central angle showing the given rotation.**

1.  $225^\circ$

2.  $-\frac{5\pi}{2}$  radians

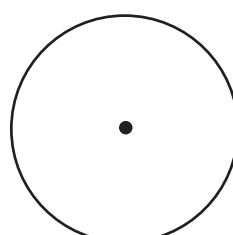
a. \_\_\_\_\_

b.



a. \_\_\_\_\_

b.

3. Give two other degree measures, one positive and one negative, for a rotation of  $138^\circ$ .4. Give two other radian measures, one positive and one negative, for a rotation of  $\frac{4\pi}{3}$ .

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**In 5–7, convert to a radian measure without using a calculator.**

5.  $60^\circ$  \_\_\_\_\_

6.  $135^\circ$  \_\_\_\_\_

7.  $-210^\circ$  \_\_\_\_\_

**In 8–10, convert to a degree measure without using a calculator.**

8.  $\frac{11\pi}{6}$  \_\_\_\_\_

9.  $-\frac{\pi}{10}$  \_\_\_\_\_

10. 3.14159 \_\_\_\_\_

**In 11–14, use a calculator to convert the given angle measure to the indicated units. Give your answer correct to the nearest thousandth.**

11.  $-42^\circ$  a. to revolutions

b. to radians

\_\_\_\_\_

\_\_\_\_\_

12.  $19\pi$  a. to revolutions

b. to degrees

\_\_\_\_\_

\_\_\_\_\_

13. 19 a. to revolutions

b. to degrees

\_\_\_\_\_

\_\_\_\_\_

14. 0.33 revolution clockwise

a. to radians

b. to degrees

\_\_\_\_\_

\_\_\_\_\_

**LESSON  
MASTER****4-2****Questions on SPUR Objectives**

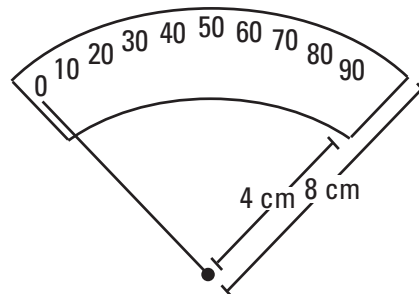
See pages 303–307 for objectives.

**Skills** Objective B

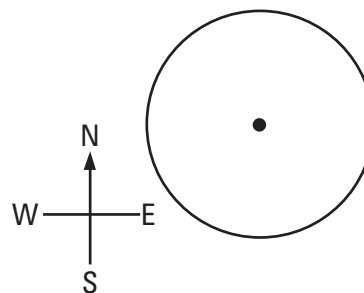
- Find the length of an arc of a circle of radius 8 m if the central angle of the arc is  $\frac{5\pi}{6}$ . \_\_\_\_\_
- Find the area of a sector of a circle of diameter 22 in. if the central angle of the sector is  $315^\circ$ . \_\_\_\_\_
- The arc of a circle of radius 4 cm has a length of  $\frac{2\pi}{3}$  cm. Find the measure of the central angle in radians and degrees. \_\_\_\_\_
- A sector in a circle with central angle  $\frac{7\pi}{12}$  has an area of  $14\pi \text{ m}^2$ . Find the exact length of the radius of the circle. \_\_\_\_\_

**Uses** Objective G

- James needs to replace the glass of the speedometer on his old car. If the needle can maximally rotate  $\frac{5\pi}{12}$ , find the area of the glass that James needs.  
\_\_\_\_\_



- Austin, TX, and Oklahoma City, OK, have approximately the same longitude,  $97^\circ 30' \text{ W}$ . Austin has latitude  $30^\circ 16' \text{ N}$ . Oklahoma City has latitude  $35^\circ 28' \text{ N}$ . Use 3,960 miles for the radius of the earth to estimate the air distance from Austin to Oklahoma city.  
\_\_\_\_\_
- Kaitlin watched her son Dizzy ride a horse 22 ft from the center of a merry-go-round. Dizzy completed one revolution in 45 seconds.
  - How far did Dizzy travel in one revolution? \_\_\_\_\_
  - How far did Dizzy travel in one minute? \_\_\_\_\_
  - Kaitlin noted that her son started at the easternmost position. If the merry-go-round rotates counterclockwise and the ride lasts 4 minutes, sketch the position of her son when the ride ended.



**LESSON  
MASTER****4-3****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Skills** Objective C**In 1–6, give the exact value of each without using a calculator.**

1.  $\cos(-90^\circ)$  \_\_\_\_\_ 2.  $\sin(-90^\circ)$  \_\_\_\_\_ 3.  $\cos 6\pi$  \_\_\_\_\_

4.  $\tan \frac{\pi}{2}$  \_\_\_\_\_ 5.  $\sin \pi$  \_\_\_\_\_ 6.  $\tan 1260^\circ$  \_\_\_\_\_

**In 7–9, use a calculator to evaluate to the nearest thousandth.**

7.  $\sin 112^\circ$  \_\_\_\_\_ 8.  $\tan \frac{\pi}{4}$  \_\_\_\_\_ 9.  $\cos(-16^\circ)$  \_\_\_\_\_

10. The point  $(1, 0)$  is rotated  $\frac{5\pi}{4}$  about the origin.

Find the coordinates of its image correct to three decimal places. \_\_\_\_\_

**Properties** Objective D11. *True or false.* For some integer values of  $k$ ,

$\tan\left(k \cdot \frac{\pi}{2}\right) = 1$ . Justify your answer.

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\_\_\_\_\_

\_\_\_\_\_

**In 12–15, describe an interval between 0 and  $2\pi$  in which  $\theta$  satisfies the given requirements.**

12.  $\cos \theta > 0$  and  $\sin \theta < 0$

\_\_\_\_\_

13.  $\sin \theta > 0$  and  $\tan \theta < 0$

\_\_\_\_\_

14.  $\cos \theta = 0$  and  $\sin \theta > 0$

\_\_\_\_\_

15.  $\tan \theta > 0$  and  $\cos \theta < 0$

\_\_\_\_\_

**Representations** Objective J**In 16–20, refer to the unit circle shown.  
Which letter best represents the value given?**

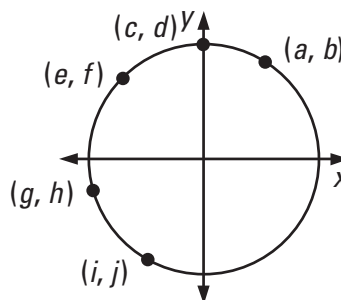
16.  $\cos -270^\circ$  \_\_\_\_\_

17.  $\sin \frac{13\pi}{12}$  \_\_\_\_\_

18.  $\sin 135^\circ$  \_\_\_\_\_

19.  $\cos \frac{\pi}{3}$  \_\_\_\_\_

20.  $\sin -\frac{2\pi}{3}$  \_\_\_\_\_



**LESSON  
MASTER****4-4****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Properties** Objective D

1. The point  $(1, 0)$  is rotated about the origin such that  $\cos \theta = -\frac{8}{17}$ .

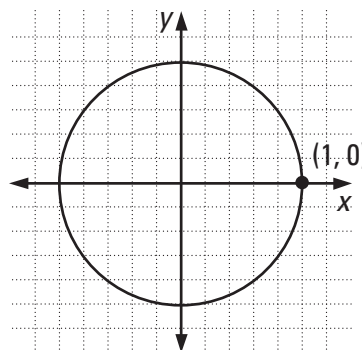
a. In what quadrant(s) could  $R_\theta(1, 0)$  lie?

\_\_\_\_\_

b. Justify your answer to part a by graphing  $R_\theta(1, 0)$  on the unit circle at the right

c. Find all possible values of  $\sin \theta$ .

\_\_\_\_\_

**Properties** Objective E

2. If  $\sin \theta = \frac{\sqrt{17}}{7}$ , find all possible values for the following.

a.  $\cos \theta$

\_\_\_\_\_

b.  $\tan \theta$

\_\_\_\_\_

3. If  $\cos \theta = 0.68$ , evaluate the following.

a.  $\cos(-\theta)$

\_\_\_\_\_

b.  $\cos(\pi - \theta)$

\_\_\_\_\_

4. If  $\sin \theta = -0.368$ , and  $\pi < \theta < \frac{3\pi}{2}$ , evaluate the following.

a.  $\sin(\pi + \theta)$

\_\_\_\_\_

b.  $\sin\left(\frac{\pi}{2} - \theta\right)$

\_\_\_\_\_

c.  $\cos(-\theta)$

\_\_\_\_\_

d.  $\tan(\pi - \theta)$

\_\_\_\_\_

5. *True or false.*  $\tan(k \cdot \pi + \theta) = \tan \theta$  for all integers  $k$ . Justify your answer.
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**LESSON  
MASTER****4-5****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Skills** Objective C

In 1–8, give the exact value.

1.  $\cos\left(-\frac{\pi}{3}\right)$  \_\_\_\_\_

2.  $\sin\left(\frac{3\pi}{4}\right)$  \_\_\_\_\_

3.  $\sin 30^\circ$  \_\_\_\_\_

4.  $\tan\left(\frac{2\pi}{3}\right)$  \_\_\_\_\_

5.  $\cos 240^\circ$  \_\_\_\_\_

6.  $\tan\left(-\frac{\pi}{4}\right)$  \_\_\_\_\_

7.  $\cos\left(\frac{17\pi}{6}\right)$  \_\_\_\_\_

8.  $\sin(-2115^\circ)$  \_\_\_\_\_

**Properties** Objective E

9. Find all values of  $\theta$  between 0 and  $2\pi$  such that  $\cos \theta = \frac{\sqrt{3}}{2}$ . \_\_\_\_\_

10. Find four values of  $\theta$  between  $-2\pi$  and  $2\pi$  such that  $\cos \theta = -\frac{1}{2}$ . \_\_\_\_\_

11. Give all possible values for  $\cos\left(k \cdot \frac{\pi}{6}\right)$  or  $\sin\left(k \cdot \frac{\pi}{6}\right)$  for all integers  $k$ . \_\_\_\_\_

12. Give all possible values for  $\cos\left(k \cdot \frac{\pi}{4}\right)$  or  $\sin\left(k \cdot \frac{\pi}{4}\right)$  for all integers  $k$ . \_\_\_\_\_

**LESSON  
MASTER****4-6****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Properties** Objective D

1. Complete the following table.

	$f(\theta) = \sin \theta$	$g(\theta) = \cos \theta$	$h(\theta) = \tan \theta$
<b>Domain</b>			
<b>Range</b>			
<b>Zeros</b>			
<b>Period</b>			
<b>Even, odd, or neither</b>			

2. For what values of  $x$  between 0 and  $2\pi$  are both  $\cos x$  and  $\tan x$  negative? \_\_\_\_\_
3. One solution to the equation  $\sin \theta = 0.732$  is  $\theta \approx 0.821$ . Find the three other solutions closest to this value. \_\_\_\_\_

**Representations** Objective K

In 4–11, identify which, if any, of the parent circular functions have graphs with the given characteristic.

4. symmetry with respect to the origin \_\_\_\_\_
5. symmetry with respect to the  $x$ -axis \_\_\_\_\_
6. symmetry with respect to the  $y$ -axis \_\_\_\_\_
7. vertical asymptotes \_\_\_\_\_
8. horizontal asymptotes \_\_\_\_\_
9. points of discontinuity \_\_\_\_\_
10.  $x$ -intercepts at integral multiples of  $\pi$  \_\_\_\_\_
11.  $y$ -intercept  $-1$  \_\_\_\_\_

**LESSON  
MASTER****4-7****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Properties** Objective F

In 1 and 2, a circular function is given.      a. State the period of the function.      b. State the amplitude of the function.

1.  $4y = \sin\left(\frac{x}{2}\right)$       a. \_\_\_\_\_      b. \_\_\_\_\_

2.  $\frac{y}{6} = \frac{\sin x}{4}$       a. \_\_\_\_\_      b. \_\_\_\_\_

3. Consider the image of the graph of  $y = \cos x$  under the transformation  $S(x, y) = \left(\frac{x}{4}, 5y\right)$ .

a. Find the amplitude of the image. \_\_\_\_\_

b. Find the period of the image. \_\_\_\_\_

c. Find an equation for the image under this transformation. \_\_\_\_\_

4. How many cycles does the graph of  $y = \sin 3x$  make for each cycle of the graph of  $y = \sin x$ ?

5. How many cycles does the graph of  $y = 3 \sin x$  make for each cycle of the graph of  $y = \sin 3x$ ?

**Uses** Objective H

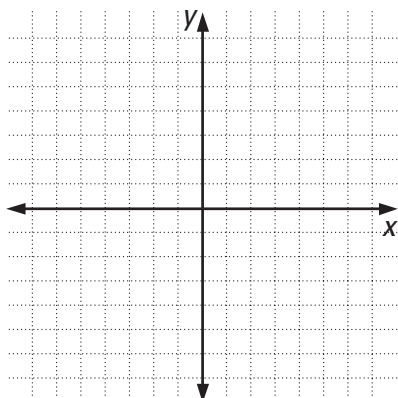
6. Suppose a tuning fork vibrates with a frequency of approximately 370 cycles per second. If the vibration displaces air molecules by a maximum of 0.22 mm, give a possible equation for the sound wave that is produced. \_\_\_\_\_

7. A certain sound wave has equation  $y = 15 \cos(110\pi t)$ . Give an equation of a sound wave with pitch one octave lower and three times as loud as this one. \_\_\_\_\_

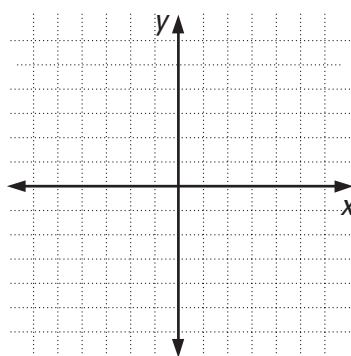
**Representations** Objective L

In 8 and 9, sketch one cycle of the graph without an automatic grapher.

8.  $y = \frac{\cos\left(\frac{x}{4}\right)}{3}$



9.  $y = 2 \sin\left(\frac{\pi}{3}x\right)$



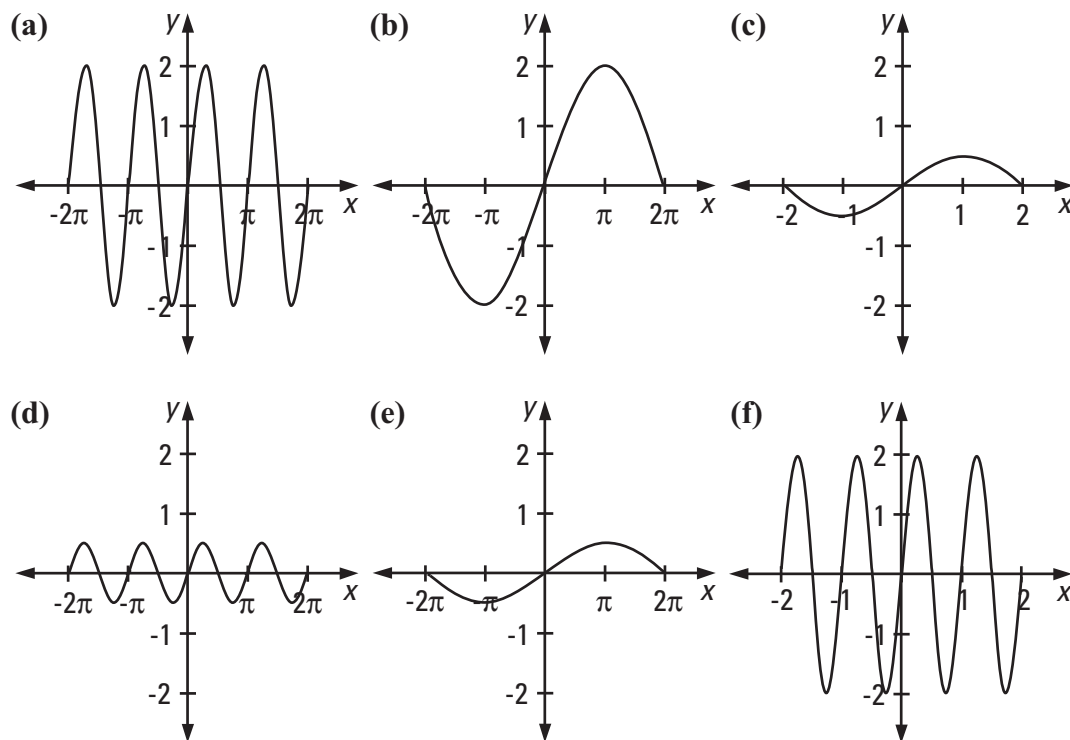
► **LESSON MASTER 4-7** page 2**Representations** Objective M

In 10–15, match each equation with its graph below.

10.  $\frac{y}{2} = \sin\left(\frac{x}{2}\right)$  \_\_\_\_\_ 11.  $2y = \sin\left(\frac{x}{2}\right)$  \_\_\_\_\_

12.  $2y = \sin 2x$  \_\_\_\_\_ 13.  $\frac{y}{2} = \sin 2x$  \_\_\_\_\_

14.  $2y = \sin\left(\frac{\pi x}{2}\right)$  \_\_\_\_\_ 15.  $\frac{y}{2} = \sin 2\pi x$  \_\_\_\_\_





**LESSON  
MASTER****4-8****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Properties** Objective F

1. Consider the graph of the function  $f(x) = \cos\left(x - \frac{9\pi}{4}\right) - 4$ .  
Find each of the following for  $f$ .

- a. the phase shift \_\_\_\_\_ b. the period \_\_\_\_\_  
c. the amplitude \_\_\_\_\_  
d. the maximum and minimum values \_\_\_\_\_

2. *True or false.* The graphs of the functions  $g(x) = \sin(x + \pi)$  and  $h(x) = \sin(x - \pi)$  are identical. Justify your answer.

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**Uses** Objective H

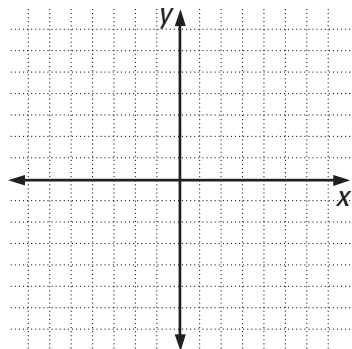
3. For an electrical-power supply, the output potential (in volts) and current (in amps) as functions of time (in seconds) are given by  $V = 25 \cos t + 25$  and  $I = 0.3 \cos\left(t - \frac{5\pi}{4}\right)$ , respectively.

- a. What are the maximum and minimum output voltages?  
\_\_\_\_\_
- b. What are the maximum and minimum output currents?  
\_\_\_\_\_
- c. What is the phase shift between output current and output voltage?  
\_\_\_\_\_
- d. By about how many seconds does the maximum current lag behind the maximum voltage?  
\_\_\_\_\_

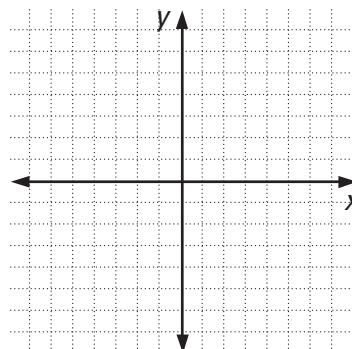
**Representations** Objective L

In 4 and 5, sketch a graph of the function.

4.  $f(x) = \sin\left(x + \frac{\pi}{2}\right) - 3$



5.  $y - 2 = \tan\left(x - \frac{\pi}{3}\right)$



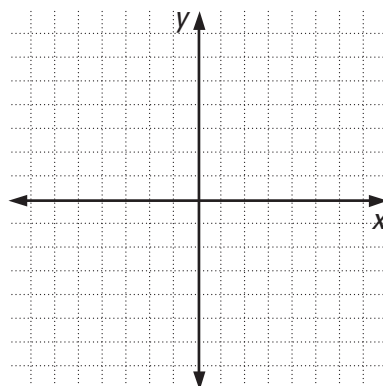
► **LESSON MASTER 4-8** page 2

6. Consider the translation

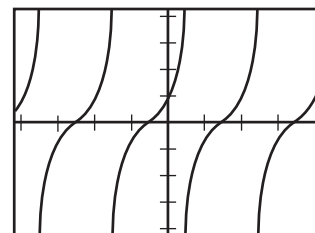
$$T: (x, y) \rightarrow \left(x - \frac{3\pi}{2}, y - 1\right).$$

- a. Graph the image of the cosine function under  $T$ .
- b. Write an equation for the image of  $y = \cos x$  under  $T$ .

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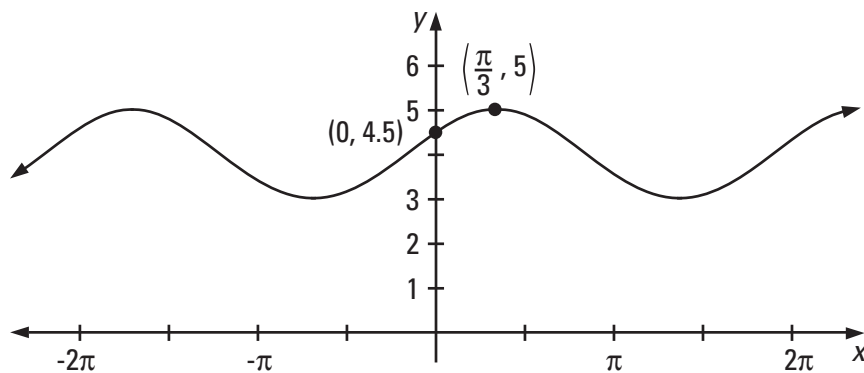
**Representations** Objective M7. Write an equation for the translation image of  $y = \tan x$  shown at the right.

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$$\begin{aligned} -2\pi \leq x \leq 2\pi & \quad x\text{-scale} = \frac{\pi}{2} \\ -4 \leq y \leq 4 & \quad y\text{-scale} = 1 \end{aligned}$$

In 8 and 9, use the designated parent function to write an equation for its translation image graphed below.

8. parent function:  $y = \cos x$ 

\_\_\_\_\_

9. parent function:  $y = \cos x$ 

\_\_\_\_\_

**LESSON  
MASTER****4-9****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Properties** Objective F**In 1 and 2, a circular function is described.****a. State the amplitude.**      **b. State the period.****c. State the phase shift.**

1.  $y = 2 \sin\left(\frac{x-4}{4}\right)$

a. \_\_\_\_\_

b. \_\_\_\_\_

c. \_\_\_\_\_

2.  $y = \cos\left(\frac{2x+\pi}{3}\right) - 4$

a. \_\_\_\_\_

b. \_\_\_\_\_

c. \_\_\_\_\_

3. Describe a scale change  $S$  and translation  $T$  whose composite maps the graph of  $y = \sin x$  onto the graph of  $y = 3 \sin\left(2x - \frac{\pi}{2}\right) + 1$ .

4. Suppose the rubber band transformation  $B: (x, y) \rightarrow \left(\frac{x-a}{h}, \frac{y}{k}\right)$

is applied to the graph of  $y = \cos x$ .

a. State an equation for the image. \_\_\_\_\_

b. Find the amplitude, period, phase shift, and vertical shift of the image.

**In 5 and 6, write a function whose graph will have the given characteristics.**

5. parent  $y = \sin x$ , phase shift  $\frac{\pi}{6}$ , period  $\pi$ ,  
amplitude  $\frac{1}{2}$

\_\_\_\_\_

6. parent  $y = \cos x$  phase shift  $\pi$ , period  $\frac{\pi}{3}$ ,  
amplitude 4

\_\_\_\_\_

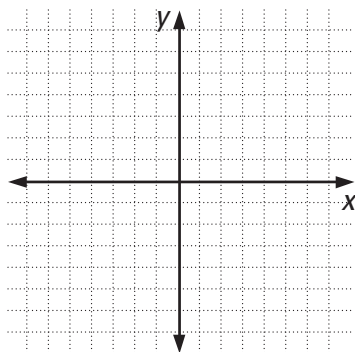
► **LESSON MASTER 4-9** page 2**Uses** Objective H

7. Dizzy, who loves the rides, is positioned on a Ferris wheel such that once the ride begins, his height  $d$  (in feet) above the ground after  $t$  seconds is given by  $d = 65 + 60 \cos\left(\frac{\pi}{90}(t - 30)\right)$ .
- |  |   |
|--|---|
| <p>a. At the start of the ride, how high is Dizzy off the ground?</p> <p>_____</p> | <p>b. How long does it take Dizzy to make one complete revolution?</p> <p>_____</p>                             |
| <p>c. What is Dizzy's maximum height above the ground?</p> <p>_____</p>            | <p>d. If the ride lasts for 9 minutes, find all times at which Dizzy is at his maximum height.</p> <p>_____</p> |

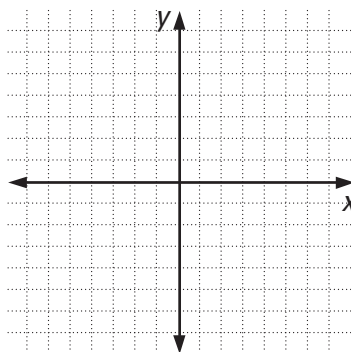
**Representations** Objective L

In 8 and 9, sketch a graph of the function described.

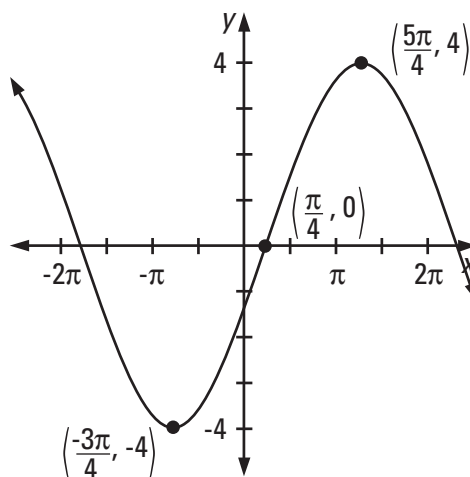
8.  $y = 2 \cos(x + \pi) - 1$



9.  $\frac{y - 2}{3} = \sin\left(4x - \frac{\pi}{6}\right)$

**Representations** Objective M

10. Give an equation for the sine wave at the right.
- \_\_\_\_\_

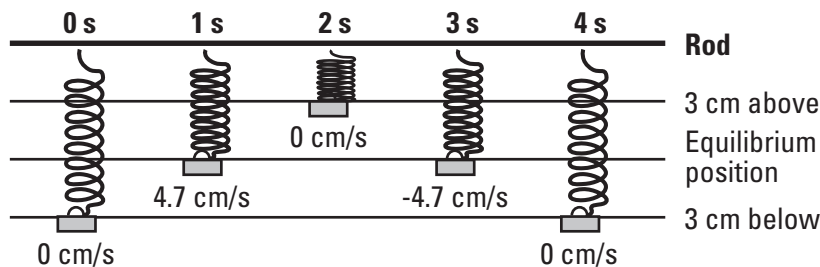


**LESSON  
MASTER****4-10****Questions on SPUR Objectives**

See pages 303–307 for objectives.

**Uses** Objective I

1. When a weight suspended from a spring is caused to oscillate, not only is its distance from equilibrium a sinusoidal function of time, but so is its velocity. The diagram below shows the velocity of the weight in cm/s at 1-second intervals for the same oscillating spring as pictured in Lesson 4-10. (Note: A positive velocity means that the weight is moving up, and a negative velocity means that the weight is moving down.)



Suppose the velocity  $v$  of the weight as a function of time  $t$  is to be modeled by an equation of the form  $\frac{v - k}{b} = \cos\left(\frac{t - h}{a}\right)$ .

- Find the amplitude of the velocity. \_\_\_\_\_
  - Find the period of the velocity. \_\_\_\_\_
  - Write an equation for the velocity function. \_\_\_\_\_
  - What is the weight's velocity at  $t = 3.5s$ ? \_\_\_\_\_
2. The following table gives the average normal temperatures in °F for six months of a year for Dodge City, KS.

Month	Jan	Feb	May	July	Aug	Nov
Average temperature (°F)	30	35	64	80	78	43

- Plot these data using the month (Jan = 1, Feb = 2, and so on) as the independent variable.
- Determine an equation for a periodic function to model these data.  
\_\_\_\_\_
- What does your model predict as the average normal temperature for September in Dodge City?  
\_\_\_\_\_

