

Multiple Choice. Match the sentence to the best description of it.

1. $x^2 + 3x + 4 = 0$.

[A] existential statement

[B] statement

[C] universal statement

[D] none of these

2. $3(x^2 + 4x + 8) = 3x^2 + 12x + 24$.

[A] universal statement

[B] existential statement

[C] statement

[D] none of these

3. *The capital of Illinois is Springfield.*

[A] statement

[B] existential statement

[C] universal statement

[D] none of these

Rewrite the expression using *and* or *or*.

4. $|x - 5| > 3$

5. $|2x + 1| < 7$

Given the statement, write an equivalent statement using the universal or existential quantifier.

6. *Some triangles are isosceles.*

7. *If n is an odd positive integer, then n^2 is also an odd positive integer.*

Identify the statement that is logically equivalent to the given statement.

8. *Some quadrilaterals are both rectangles and squares.*

[A] *No quadrilateral that is a rectangle can also be a square.*

[B] *There exists a quadrilateral that is a rectangle and a square.*

[C] *No quadrilateral is both a rectangle and a square.*

[D] None of these is equivalent to the original.

Identify the statement that is logically equivalent to the given statement.

9. *No equilateral triangle is a right triangle.*

[A] *There exists a triangle that is both equilateral and right.*

[B] *No triangle that is equilateral can also be right.*

[C] *Every right triangle is equilateral.*

[D] None of these is equivalent to the original.

Write the negation of the statement:

10. *All mammals breathe air and some can swim.*

11. *Some people drink coffee or all people eat chocolate.*

12. *Some rhombuses are squares.*

Determine the truth value of the given statement. Justify your answer.

13. \exists a real number x such that $x = \frac{1}{x}$.

14. \exists real numbers x and y with $x < 0$ and $y < 0$ such that $xy < 0$.

15. $\forall x \in \mathbf{R}, x^2 > 0$.

16. Let $s(n)$ be the sentence $n^2 < 2n + 7$. What is the truth value of $s(4)$?

Indicate whether the argument is valid or invalid, and match it to the appropriate argument form or error.

17. $\left\{ \begin{array}{l} \text{If a number } n \text{ greater than } 2 \text{ is prime, then } n \text{ is odd.} \\ 24 \text{ is even.} \end{array} \right.$
 $\therefore 24 \text{ is not prime.}$

[A] valid; Law of Transitivity

[B] valid; Law of Detachment

[C] valid; Law of Indirect Reasoning

[D] invalid; Inverse Error

[E] invalid; Converse Error

[F] invalid; Improper Induction

Indicate whether the argument is valid or invalid, and match it to the appropriate argument form or error.

18. $\begin{cases} \text{If } \triangle ABC \cong \triangle RST, \text{ then } \overline{AC} \cong \overline{RT}. \\ \overline{AC} \cong \overline{RT}. \end{cases}$
 $\therefore \triangle ABC \cong \triangle RST.$

[A] valid; Law of Detachment

[B] invalid; Improper Induction

[C] invalid; Inverse Error

[D] invalid; Converse Error

[E] valid; Law of Indirect Reasoning

[F] valid; Law of Transitivity

19. $\begin{cases} \text{If } \theta \text{ is in Quadrant II, then } \tan \theta < 0. \\ \theta \text{ is not in Quadrant II.} \end{cases}$
 $\therefore \tan \theta \geq 0.$

[A] invalid; Inverse Error

[B] valid; Law of Transitivity

[C] invalid; Converse Error

[D] valid; Law of Detachment

[E] valid; Law of Indirect Reasoning

[F] invalid; Improper Induction

20. $1^2 < 5(1)$
 $2^2 < 5(2)$
 $3^2 < 5(3)$
 $4^2 < 5(4)$
 $\therefore \forall \text{ integer } n, n^2 < 5n.$

[A] invalid; Inverse Error

[B] valid; Law of Detachment

[C] valid; Law of Indirect Reasoning

[D] invalid; Converse Error

[E] invalid; Improper Induction

[F] valid; Law of Transitivity

Finish the statement and supply a two-column proof.

21. If $2x + 34 = 2(x \pm 3)^2$, then $x = \underline{\hspace{1cm}}?$ or $x = \underline{\hspace{1cm}}?$.

22. If $6x + 18 = 3(x \pm 1)^2$, then $x = \underline{\hspace{1cm}}?$ or $x = \underline{\hspace{1cm}}?$.

Deduce a valid conclusion using all of the given premises. You may need to write the contrapositive of one or more of the statements, and you may need to reorder them.

23. $\left\{ \begin{array}{l} \text{Melinda went to college.} \\ \text{If you take math, you will be smart.} \\ \text{If you do not take math, you cannot go to college.} \\ \text{If you are smart, you will be happy.} \end{array} \right.$
24. $\left\{ \begin{array}{l} \text{If you read news online, you will be well informed.} \\ \text{If you are well informed, you will have many friends.} \\ \text{Rick does not have many friends.} \end{array} \right.$

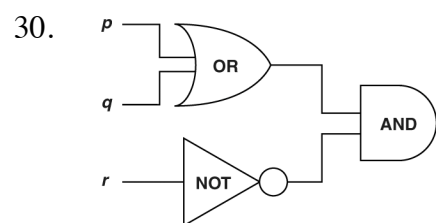
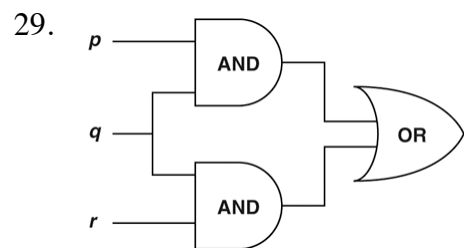
True or False

25. For all children c , c is able to swim.
26. There exists a country c such that c 's basic unit of currency is the yen.

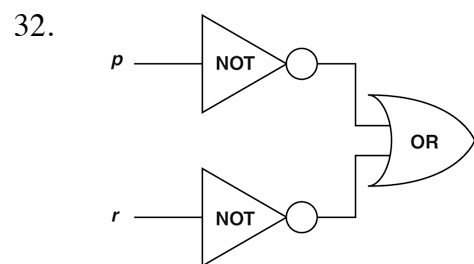
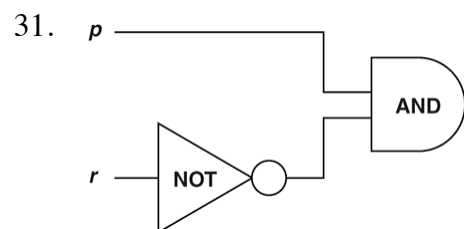
Indicate whether the argument is valid or invalid, and match it to the appropriate argument form or error.

27. $\left\{ \begin{array}{l} \text{If you study hard, you will get good grades.} \\ \text{If you get good grades, you will get a good job.} \\ \therefore \text{If you study hard, you will get a good job.} \end{array} \right.$
- | | |
|---------------------------------|--------------------------------------|
| [A] invalid; Converse Error | [B] valid; Law of Transitivity |
| [C] invalid; Improper Induction | [D] valid; Law of Indirect Reasoning |
| [E] valid; Law of Detachment | [F] invalid; Inverse Error |
28. $\left\{ \begin{array}{l} \text{If you pass Precalculus with a grade of C or better, you may enroll in Calculus.} \\ \text{Shana got an A in Precalculus.} \\ \therefore \text{Shana may enroll in Calculus.} \end{array} \right.$
- | | |
|--------------------------------------|--------------------------------|
| [A] valid; Law of Indirect Reasoning | [B] valid; Law of Transitivity |
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Write the logical expression that corresponds to the network.



Write an input-output table for the network.



Write a truth table for the expression.

33. $(\text{not } (p \text{ or } q)) \text{ or } (p \text{ and } q)$

Write a truth table for the expression.

34. $(p \text{ and } q) \text{ or } (p \text{ and not } q)$