Name

LESSON MASTER 9-7

Questions on SPUR Objectives
See pages 622–625 for objectives.

Properties Objective F

1. Given $p(z) = (z - \sqrt{7})(z + \sqrt{7})(z - (1 + 2i))^2(z + (\text{-}1 + 2i))^2(z - 4)^3$, determine the zero(s) of p with each multiplicity.

 a. one $\sqrt{7}, \text{-}\sqrt{7}$

 b. two $1 + 2i, 1 - 2i$

 c. three 4

Properties Objective H

2. Find all solutions to $x^4 - 2x^3 + 2i^2 - 10x + 25 = 0$, given that one solution is $x = 2 + i$.
 $x = 2 + i, 2 - i, \text{-}1 + 2i, \text{-}1 - 2i$

In 3–5, *true or false*. Justify your answer.

3. The equation $p(x) = x^8 - 1$ has eight complex zeros.
 True; Number of Zeros of a Polynomial Theorem

4. Every polynomial function with real coefficients that has a zero $3 + 2i$ also has a zero $3 - 2i$.
 True; Conjugate Zeros Theorem

5. It is possible for the graph of a third-degree polynomial with real coefficients to cross the line $y = 4$ exactly twice.
 False; $p(x) - 4 = 0$ is a polynomial with real coefficients. By the Conjugate Zeros Theorem, it has an even number of nonreal zeros.

Representations Objective K

In 6–8, a function f is given. a. Graph $f(x)$ to find all the real zeros. b. Factor $f(x)$ to determine all the nonreal zeros.

6. $f(x) = x^3 - 4x^2 + 9x - 10$
 a. 2
 b. $1 + 2i, 1 - 2i$

7. $f(x) = x^4 - 8x^3 + 23x^2 - 30x + 18$
 a. 3
 b. $1 + i, 1 - i$

8. $f(x) = x^5 + 19x^4 + 140x^3 + 506x^2 + 924x + 720$
 a. $\text{-}4, \text{-}5, \text{-}6$
 b. $\text{-}2 + \sqrt{2}i, \text{-}2 - \sqrt{2}i$

Name

LESSON MASTER 9-8

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

1. Find the four fourth roots of 64.
 $2\sqrt{2}, \text{-}2\sqrt{2}, 2\sqrt{2}i, \text{-}2\sqrt{2}i$

2. Find the three cube roots of -1.
 $\text{-}1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

In 3–10, factor the given polynomial completely over the set of polynomials with integer coefficients.

3. $9 - 16x^2$ $(3 + 4x)(3 - 4x)$
4. $125n^3 - 27m^3$ $(5n - 3m)(25n^2 + 15nm + 9m^2)$
5. $x^6 - 512$ $(x^2 - 8)(x^4 + 8x^2 + 64)$
6. $u^8 - v^8$ $(u + v)(u - v)(u^2 + v^2)(u^4 + v^4)$
7. $343x^3y^3 + 1$ $(7xy + 1)(49x^2y^2 - 7xy + 1)$
8. $t^6 - 729$ $(t + 3)(t - 3)(t^2 + 3t + 9)(t^2 - 3t + 9)$
9. $m^5 + 216m^2$ $m^2(m + 6)(m^2 - 6m + 36)$
10. $32w^3 - 4$ $4(2w - 1)(4w^2 + 2w + 1)$

In 11–15, factor the given polynomial completely over the set of polynomials with rational coefficients.

11. $a^5b^5 + c^5$
 $(ab + c)(a^4b^4 - a^3b^3c + a^2b^2c^2 - abc^3 + c^4)$

12. $c^7 + 128$
 $(c + 2)(c^6 - 2c^5 + 4c^4 - 8c^3 + 16c^2 - 32c + 64)$

13. $\text{-}r^5 - t^{10}$
 $(r + t^2)(\text{-}r^4 + r^3t^2 - r^2t^4 + rt^6 - t^8)$

14. $\frac{d^9}{512} + 1$
 $\frac{1}{512}(d + 2)(d^2 - 2d + 4)(d^6 - 8d^3 + 64)$

15. $g^6 - g$
 $g(g - 1)(g^4 + g^3 + g^2 + g + 1)$

Name

LESSON MASTER 9-9

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

In 1–4, factor the expression completely over the set of polynomials with integer coefficients.

1. $4r^3 - 6r^2 - 6r + 9$ $(2r - 3)(2r^2 - 3)$
2. $3s^5 - 12s^3 - s^2 + 4$ $(s + 2)(s - 2)(3s^3 - 1)$
3. $2x^3 + 12x^2 - 5x - 30$ $(x + 6)(2x^2 - 5)$
4. $u^4 + 5u^3 - u - 5$ $(u - 1)(u + 5)(u^2 + u + 1)$

In 5–8, find all solutions.

5. $y^3 + 8y^2 - 3y = 24$ $y = \text{-}8, y = \text{-}\sqrt{3}, y = \sqrt{3}$
6. $27m^3 - 18m^2 = 12m - 8$ $m = \text{-}\frac{2}{3}, m = \frac{2}{3}$
7. $20x^3 - 4x^2 - 25x = \text{-}5$ $x = \frac{1}{5}, x = \frac{\sqrt{5}}{2}, x = \text{-}\frac{\sqrt{5}}{2}$
8. $n^6 - n^4 = n^2 - 1$ $n = \text{-}1, n = 1, n = \text{-}i, n = i$

In 9–14, factor the expression completely over the set of polynomials with integer coefficients.

9. $6x^2 + 3xy + 2x + y$ $(3x + 1)(2x + y)$
10. $3a^4 - 5a^2b - 2b^2$ $(3a^2 + b)(a^2 - 2b)$
11. $x^3w^2 - x^3z^2 - y^3w^2 + y^3z^2$ $(w + z)(w - z)(x - y)(x^2 + xy + y^2)$
12. $a^4 + a^2d + a^2b + bd$ $(a^2 + d)(a^2 + b)$
13. $12x^5 + 4x^4y - 3x^3 - x^2y$ $x^2(2x + 1)(2x - 1)(3x + y)$
14. $2n^3 + n^2m + 2nm + m^2 + 6n + 3m$ $(2n + m)(n^2 + m + 3)$

In 15–17, solve for x.

15. $4x^2 + 11xy - 3y^2 = 0$ $x = \frac{y}{4}, x = \text{-}3y$
16. $24x^2 - 18xw = 20x - 15w$ $x = \frac{5}{6}, x = \frac{3w}{4}$
17. $3x^3 - 3x^2 - 2ax^2 + 2ax = 0$ $x = 0, x = 1, x = \frac{2a}{3}$

Name

LESSON MASTER 10-1

Questions on SPUR Objectives
See pages 692–695 for objectives.

Properties Objective C

1. Consider the binomial distribution function B with $n = 10$ and $p = 0.83$.

 a. Give a formula for $B(k)$.
 $B(k) = {}_{10}C_k \cdot 0.83^k \cdot 0.17^{10-k}$

 b. Give the function's domain.
 $0 \le k \le 10$ for integers k

 c. Give the distribution's mode.
 9

 d. Give the function's maximum.
 ≈ 0.32

 e. Describe how the domain and maximum value change if $n = 100$ and $p = 0.83$.
 Domain increases; maximum value decreases.

2. Show that for a binomial distribution B with a fixed number of trials n and a fixed probability $p = 0.5$, $B(k) = B(n - k)$.
 $B(k) = {}_nC_k \cdot 0.5^k \cdot 0.5^{n-k} = {}_nC_k \cdot 0.5^n$; $B(n - k) = {}_nC_{n-k} \cdot 0.5^{n-k} \cdot 0.5^{n-(n-k)} = {}_nC_{n-k} \cdot 0.5^n$. Since ${}_nC_k = {}_nC_{n-k}$, $B(k) = B(n - k)$.

Uses Objective E

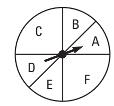
3. Suppose the Food and Drug Administration is testing a new prescription medication which the manufacturer claims has a success rate of 60%. Assume this success rate and find the probability that at least 60% of the subjects given the medication will respond positively for the given number of people.

 a. 10 people $\approx 63.3\%$

 b. 20 people $\approx 59.6\%$

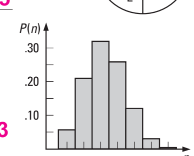
Representations Objective I

4. Consider the spinner pictured at the right. Sectors A, B, D, and E have central angles of 45°, while sectors C and F have central angles of 90°.



 a. If the spinner is spun once, what is the probability that it will land in either sector A or sector C?
 0.375

 b. Suppose the spinner is spun 6 times. Construct a histogram for the probability distribution P, where $P(n)$ is the probability that the spinner lands in sector A or sector C n times.
 Bar heights: 0.06, 0.21, 0.32, 0.26, 0.12, 0.03, 0.003



Name

LESSON MASTER 10-2 **Questions on SPUR Objectives** See pages 692–695 for objectives.

Skills Objective A

In 1–4, a binomial experiment is described.
a. Find the mean number of successes.
b. Find the standard deviation for the number of successes.

1. The number of trials is 30 and the probability of success on each trial is 0.7. a. 21 b. ≈ 2.5
2. The number of trials is 100 and the probability of failure on each trial is 0.85. a. 15 b. ≈ 3.57
3. A student guesses randomly on a test with 50 true-false questions. a. 25 b. ≈ 3.54
4. Two fair dice are tossed 180 times. A success is tossing a 7. a. 30 b. 5

Uses Objective E

5. A manufacturer of spark plugs has estimated that the probability that a spark plug will be defective is 0.0125. A trucking company recently bought ten gross (1440) of these spark plugs for their fleet.
   a. About how many spark plugs should the trucking company expect to be defective? 18 spark plugs
   b. What is the probability that the trucking company will find exactly the expected number of defective spark plugs? ≈ 0.094
6. Suppose you flip a quarter 16 times and count the times it lands heads up.
   a. What is the expected number of heads? 8 heads
   b. What is the probability that the number of heads is no more than one standard deviation from the expected number? ≈ 0.79
7. Suppose the following experiment is conducted: A die with s sides marked 1 through s is tossed n times and the number of times a 1 is tossed is recorded. After many repetitions of the experiment, it is found that the number of 1s tossed has a mean of 33 and a standard deviation of 5.5.
   a. If the die is assumed to be fair, what is the most likely number of sides it has? 12 sides
   b. If the die is assumed to be fair, how many times n was it tossed in each experiment? 396 times

97

Name

LESSON MASTER 10-3 **Questions on SPUR Objectives** See pages 692–695 for objectives.

Skills Objective F Sample answers are given.

1. Roland and Diane are playing a board game with a 6-sided die. Diane boasts that she has a lucky way of tossing the die which will give her a 6 more often than normal. Roland does not believe her, so he asks Diane to toss the die 12 times. She does so and gets five 6s.
   a. State a null and an alternative hypothesis for testing Diane's claim.
   H_0: Diane has no effect on the outcomes.
   H_1: Diane has an effect on the outcomes.
   b. Can your null hypothesis be rejected at the 0.05 significance level? Justify your answer.
   Yes; the probability of tossing five or more 6s in 12 times is $\approx 0.036 \le 0.05$. So the null hypothesis can be rejected at the 0.05 level.
   c. Roland, still doubting Diane's luck, does a test to see if the die is biased. He tosses it 12 times and gets three 6s. Test the claim that the die is biased at the 0.05 significance level. Be sure to clearly state your hypotheses.
   H_0: The die is unbiased toward 6s. H_1: The die is biased toward 6s. The probability of tossing three or more 6s in 12 times is $\approx 0.32 > 0.05$. So the null hypothesis cannot be rejected.
2. A thermometer manufacturer claims that at least 95% of its thermometers are accurate to within 0.1°C.
   a. State a null and an alternative hypothesis for testing the manufacturer's claim. H_0: At least 95% of the thermometers are accurate. H_1: Fewer than 95% of the thermometers are accurate.
   b. Suppose that of 20 thermometers randomly selected and checked for accuracy, five are found to give temperature readings which are off by more than 0.1°C. Test the manufacturer's claim at the 0.01 significance level.
   The probability that five or more thermometers are off by more than 0.1°C is $\approx 0.0026 < 0.01$. So, the claim can be rejected at the 0.01 level.

98

Name

LESSON MASTER 10-4 **Questions on SPUR Objectives** See pages 692–695 for objectives.

Vocabulary

1. What are the *inflection points* of the graph of the sine function? $(n\pi, 0)$, for integers n

Properties Objective D

2. Under what condition(s) does the graph of a binomial distribution approach that of a normal curve? The graph approaches a normal curve as the number of trials, n, increases.

In 3–7, ***true or false.***

3. The parent and standard normal functions have the same domain. True
4. The parent and standard normal functions have the same range. False
5. The areas under the parent and standard normal curves are the same. False
6. The parent and standard normal curves have the same inflection points. False
7. The scale-change transformation $S: (x, y) \to \left(\sqrt{2\pi}\,x, \frac{y}{\sqrt{2}}\right)$ maps the parent normal curve onto the standard normal curve. False

In 8–11, let $f(x) = e^{-x^2}$ and $g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$.

8. $f(0)$ 1
9. $g(0)$ $\frac{1}{\sqrt{2\pi}} \approx 0.40$
10. $\lim\limits_{x\to\infty} f(x)$ 0
11. $\lim\limits_{z\to\infty} g(z)$ 0
12. What is the median of a standard normal distribution? 0
13. Consider the graph of the normal distribution function given by $h(x) = \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{3}}$.
   a. Find the graph's inflection points. $\left(\frac{\sqrt{6}}{2}, \frac{1}{\sqrt{e\pi}}\right), \left(-\frac{\sqrt{6}}{2}, \frac{1}{\sqrt{e\pi}}\right)$
   b. Find the area between the graph and the x-axis. $\sqrt{3}$

99

Name

LESSON MASTER 10-5 **Questions on SPUR Objectives** See pages 692–695 for objectives.

Skills Objective B

Use the Standard Normal Distribution Table in Appendix D.

1. a. Find the probability that a standard normal random variable z will take on a value between 0.05 and -1.02. 0.366
   b. Determine the area under the standard normal curve from $z = -1.02$ to $z = 0.05$. 0.366
2. About what percent of the data in a standard normal distribution are within 1.2 standard deviations of the mean? $\approx 77\%$

In 3–6, evaluate the given probability.

3. $P(z > 1.5)$ 0.668
4. $P(z < -2.24)$ 0.0125
5. $P(|z| \le 1)$ 0.6826
6. $P(-1.35 < z < 1.12)$ 0.7801

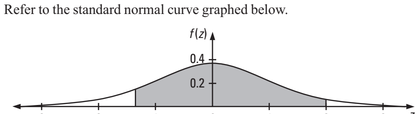
Properties Objective D

In 7–9, let z be a random variable with a standard normal distribution and let a be some constant. *True or false.*

7. $P(z > a) = 1 - P(z < a)$ True
8. $P(z = a) = 0$ True
9. $P(|z| < a) = P(z < a) - P(z > a)$ True

Representations Objective J

10. Refer to the standard normal curve graphed below.



   a. Let the area of the shaded region for $z < 0$ be 0.398. Find a. -1.27
   b. What is the probability that a standard normal random variable will be in the shaded region of the graph? 0.8752

100

Name

LESSON MASTER 10-6

Questions on SPUR Objectives
See pages 692–695 for objectives.

Properties Objective D

1. Explain why a binomial distribution with mean 16 and standard deviation 1.8 should not be approximated by a normal distribution.
Since $\mu = np = 16$ and $\sigma = \sqrt{npq} = 1.8$, $q \approx 0.20$. So $p \approx 0.8$. Thus $n \approx 20$ and $nq \approx 4 < 5$. So nq is not great enough for the binomial distribution to be approximated by a normal distribution.

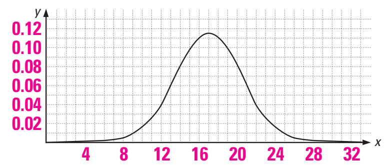
Uses Objective E

2. Suppose a global computer network is able to transmit digital data at speeds of 1.5 Mbps (megabits per second). Assume this is the mean speed and that the speed is normally distributed with a standard deviation of 0.2 Mbps. If someone needs to send a data file of 400 Mb to an office in Japan, what is the probability that the data transmission will take less than 5 minutes? **0.2033**

3. A flask is divided in half by a permeable membrane which allows atoms of helium to pass freely from either side of the flask to the other side. The flask is filled with approximately 10^6 helium atoms. Assuming that there is an equal probability that a helium atom is on either side of the membrane, find the probability that at any moment either side of the flask will contain more than 50.1% of the helium atoms. **≈ 0.0228**

Representations Objective J

4. Consider a normal probability distribution with mean 17 and standard deviation 3.5.
   a. Find an equation to model this distribution. $\frac{1}{3.5\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-17}{3.5}\right)^2}$
   b. Draw a graph of the distribution.



   c. What is the area under this graph between $x = 13.5$ and $x = 20.5$? **≈ 0.68**

Name

LESSON MASTER 10-7

Questions on SPUR Objectives
See pages 692–695 for objectives.

Uses Objective F

1. Suppose a car manufacturer claimed its new economy car has an average highway gas mileage of 37 mpg with a standard deviation of 5 mpg. After receiving numerous complaints, a consumer group decided to test a random sample of 40 such vehicles. For the sample, they found the mean gas mileage to be 35 mpg. At the 0.01 significance level, test whether the average gas mileage is less than the manufacturer claims. Be sure to state your hypotheses clearly.
H_0: The highway mileage is 37 mpg. H_1: The highway highway mileage is not 37 mpg. $\mu_{\bar{x}} = 35$ and $\sigma_{\bar{x}} = \frac{5}{\sqrt{40}} \approx 0.791$, so $\frac{\bar{x} - \mu}{\sigma} \approx -2.53$. $P(z \le -2.53) \approx 0.0057 < 1$, so H_0 can be rejected.

Uses Objective G

2. In Ohio in 1995–1996 the scores of the seniors taking the mathematics section of the Scholastic Aptitude Test (SAT-M) had a mean of 535 and a standard deviation of 104. What is the probability that a random sample of 40 of these students will have a mean SAT-M score less than 520? **≈ 0.1814**

3. a. Consider an experiment in which a fair six-sided die is tossed 400 times. Describe the distribution of the outcomes. Give the distribution's mean and standard deviation.
(Recall $\mu = \sum_{i=1}^{n} x_i \cdot P(x)$ and $\sigma^2 = \sum_{i=1}^{n} (x_i^2 \cdot P(x_i)) - \mu^2$.)
The distribution is uniform with a constant probability of $\frac{1}{6}$. The mean is 3.5; the standard deviation is ≈ 1.708.

   b. Suppose you repeat the experiment of part a many times. Describe the distribution of the means of these experiments. Give the distribution's mean and standard deviation.
The distribution of sample means is normal with a mean of 3.5 and a standard deviation of $\approx \frac{1.708}{\sqrt{400}} \approx 0.0854$.

Name

LESSON MASTER 10-8

Questions on SPUR Objectives
See pages 692–695 for objectives.

Uses Objective H

1. A military expedition using a GPS (Global Positioning System) receiver finds their altitude to be 1233 ± 28 m. If the margin of error represents the 95% confidence interval for the receiver, what is the probability that the expedition is above 1261 m? **0.025**

2. Suppose an astronomer has made many measurements of the distance to a distant star cluster and finds the data to be normally distributed with a mean of 624 kiloparsecs and a standard deviation of 35 kiloparsecs. (1 kiloparsec = 3.08×10^{19} meters)
   a. What is the 90% confidence interval for the distance to the cluster? **566 kpc ≤ μ ≤ 682 kpc**
   b. What is the 99% confidence interval for the distance to the cluster? **534 kpc ≤ μ ≤ 714 kpc**

3. A research team studying the dietary habits of American adult females charts the daily sodium intake of 100 randomly selected women over the course of 2 months. For their sample group, they find the daily sodium intake to have a mean of 2,850 mg with a standard deviation of 450 mg. Because the sample size is large, the researchers feel the standard deviation of the sample represents the standard deviation of the entire population of American women.
   a. What should the research team report as the 95% confidence interval for the mean daily sodium intake of American women? **2762 mg ≤ μ ≤ 2938 mg**
   b. If the research team had wanted to report the mean sodium intake with a 95% confidence interval of 36 mg, what should their sample size have been? **2401**

4. Standardized tests are called "standardized" because they are given to a large number of students whose scores serve as the standard scores for others. Initially, standardized tests must be given trials. Suppose a standardized test with a total score of 40 is sampled with 200 randomly selected students from a population, and there is a mean of 16.7 and a standard deviation of 6.3 for the sample.
   a. Give the 90% confidence interval for the population mean. **15.97 ≤ μ ≤ 17.43**
   b. To reduce this confidence interval to one half its size, what size sample would be needed? **806**

Name

LESSON MASTER 11-1

Questions on SPUR Objectives
See pages 749–751 for objectives.

Skills Objective A

In 1–4, find the product, if possible.

$A = \begin{bmatrix} 7 & 6 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ 2 & 7 \\ 8 & 5 \end{bmatrix}$ $C = [x \ \ y]$

1. AB **not possible**
2. BA $\begin{bmatrix} -4 & -4 \\ 35 & 26 \\ 71 & 58 \end{bmatrix}$
3. CA $\begin{bmatrix} 7x + 3y & 6x + 2y \end{bmatrix}$
4. A^2 $\begin{bmatrix} 67 & 54 \\ 27 & 22 \end{bmatrix}$

Properties Objective D

In 5–8, A is a 2×4 matrix, B is 4×3, and C is 3×4. Determine the dimensions of the indicated product matrix, if the product can be formed.

5. BC **4 × 4**
6. CB **3 × 3**
7. $A(BC)$ **2 × 4**
8. $A(CB)$ **not possible**

Uses Objective F

In 9 and 10, use the production matrix P and the cost matrix C shown here.

$$P = \begin{array}{c} \text{Motors} \quad \text{Rotors} \\ \begin{bmatrix} 10{,}000 & 15{,}000 \\ 5{,}000 & 7{,}000 \\ 6{,}000 & 6{,}000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Factory 1} \\ \text{Factory 2} \\ \text{Factory 3} \end{array} \qquad C = \begin{array}{c} \text{Cost to Produce} \quad \text{Cost to Consumer} \\ \begin{bmatrix} 50 & 150 \\ 5 & 10 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Motors} \\ \text{Rotors} \end{array}$$

9. Calculate PC; tell what the product matrix represents.
$\begin{bmatrix} 575{,}000 & 1{,}650{,}000 \\ 285{,}000 & 820{,}000 \\ 330{,}000 & 960{,}000 \end{bmatrix}$
1st col.: production costs;
2nd col.: customer costs.

10. Find the profits made by each factory before marketing and other costs, assuming that all items produced are sold.
Factory 1 **1,075,000** Factory 2 **535,000** Factory 3 **630,000**