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4-1 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 275–277 for objectives.

PROPERTIES Objective F

In 1–4, tell whether the statement is true or false and justify your answer.

1. 14 is a factor of 154.
true; $154 = 14 \cdot 11$ and 11 is an integer.

2. 10,001 is divisible by 1001.
false; There is no integer n such that $10{,}001 = 1001n$.

3. $2n$ is a factor of $4n^7 + 2n^4 + 6n$.
true; $4n^7 + 2n^4 + 6n = 2n(2n^6 + n^3 + 3)$ and $2n^6 + n^3 + 3$ is a polynomial.

4. Prove or disprove the following conjecture:

Let p be an integer. If 3 is a factor of p, then 3 is a factor of $p + 12$.

Proof: Let p be an integer and let 3 be a factor of p. Then $p = 3n$ for some integer n. So $p + 12 = 3n + 12 = 3(n + 4)$. Since $n + 4$ is an integer, 3 is a factor of $p + 12$.

5. Find a counterexample to disprove the following statement:
For all polynomials $q(x)$, $r(x)$, and $s(x)$, if $(s(x)$ is a factor of $q(x) + r(x))$, then $(q(x)$ is divisible by $s(x)$ or $r(x)$ is divisible by $s(x))$.

Answers vary. Sample. Let $q(x) = x^2$, $r(x) = -4$, and $s(x) = x - 2$. Then $q(x) + r(x) = x^2 - 4$ is divisible by $s(x)$, but $q(x)$ is not divisible by $s(x)$ and $r(x)$ is not divisible by $s(x)$.

6. Prove that the sum of any five consecutive integers is divisible by 5.

Proof: Let n be any integer. Then any set of five consecutive integers can be written as $n, n + 1, n + 2, n + 3$, and $n + 4$. Their sum is $5n + 10$. Note that $5n + 10 = 5(n + 2)$. Since $n + 2$ is an integer, $5n + 1$ is divisible by 5 by the definition of factor. Thus, the sum of any five consecutive integers is divisible by 5.

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4-2 Lesson Master

Questions on SPUR Objectives
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SKILLS Objective A

In 1–4, a pair of integers n and d are given. Find integers q and r as defined in the Quotient-Remainder Theorem.

1. $n = 42$, $d = 12$ $q = 3, r = 6$
2. $n = 186$, $d = 22$ $q = 8, r = 10$
3. $n = 3$, $d = 5$ $q = 0, r = 3$
4. $n = 34{,}998$, $d = 25$ $q = 1399, r = 23$

In 5 and 6, three of the four integers n, d, q, and r as defined in the Quotient-Remainder Theorem are given. Find the missing value.

5. $n = 1213$, $q = 7$, $r = 2$ $d = 173$
6. $d = 22$, $q = 19$, $r = 1$ $n = 419$

In 7–9, find the remainder in the given situation.

7. When $x^4 + 2x^2 + 9x - 1$ is divided by $x^3 - 5x^2$, the quotient is $x + 5$. $27x^2 + 9x - 1$
8. When $3x^5 - 2x^4 + 12$ is divided by $x^2 - 1$, the quotient is $3x^3 - 2x^2 + 3x - 2$. $3x + 10$
9. When $x^{12} + 6x^9 + x^4 - 5x + 11$ is divided by $x^4 - x + 2$, the quotient is $x^8 + 7x^5 - 2x^4 + 7x^2 - 16x - 5$. $7x^3 - 30x^2 + 32x - 1$

PROPERTIES Objective H

10. When $8x^4 + 5x + 2$ is divided by a polynomial $d(x)$, the remainder is $r(x)$. If $r(x)$ is not the zero polynomial, what are the possible degrees of $r(x)$? 0, 1, 2, or 3

11. When Shannon and Nathan applied the Quotient-Remainder Theorem to divide the polynomial $n(x)$ by the polynomial $d(x)$, they found different quotient polynomials. Can both of them be correct? Explain how you know. No; the Quotient–Remainder Theorem guarantees that $q(x)$ and $r(x)$ are unique.

USES Objective J

12. Sarah is arranging senior pictures in the yearbook. She can fit 9 photos on one page. There are 417 students in the senior class this year, and each student submits one photo.

a. How many full pages of senior photos will there be? 46 pages

b. How many photos will be on the last, partial page? 3 photos

c. Relate your answers to Parts a and b to the Quotient-Remainder Theorem.
When 417 is divided by 9, the quotient is 46 and the remainder is 3. $417 = 46 \cdot 9 + 3$

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4-3 Lesson Master

Questions on SPUR Objectives
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SKILLS Objective A

In 1–4, use long division to find the quotient $q(x)$ and the remainder $r(x)$ when $p(x)$ is divided by $d(x)$.

1. $p(x) = 2x^3 + 3x - 5$, $d(x) = x + 4$

$q(x) = 2x^2 - 8x + 35$

$r(x) = -145$

$$\begin{array}{r} 2x^2 - 8x + 35 \\ x + 4\,\overline{)\,2x^3 \qquad + 3x - 5} \\ \underline{2x^3 + 8x^2} \qquad\qquad \\ -8x^2 + 3x \qquad \\ \underline{-8x^2 - 32x} \qquad \\ 35x - 5 \\ \underline{35x + 140} \\ -145 \end{array}$$

2. $p(x) = 7x^5 - 9x^3 + 2x^2 - 5x - 7$, $d(x) = x^2 - 2$

$q(x) = 7x^3 + 5x + 2$

$r(x) = 5x - 3$

$$\begin{array}{r} 7x^3 + \qquad 5x + 2 \\ x^2 - 2\,\overline{)\,7x^5 - 9x^3 + 2x^2 - 5x - 7} \\ \underline{7x^5 - 14x^3} \qquad\qquad\qquad \\ 5x^3 + 2x^2 - 5x \qquad \\ \underline{5x^3 \qquad - 10x} \qquad \\ 2x^2 + 5x - 7 \\ \underline{2x^2 \qquad - 4} \\ 5x - 3 \end{array}$$

PROPERTIES Objective H

In 3–5, a polynomial is given. Without dividing, find the remainder when the polynomial is divided by
a. $x - 6$. b. $x + 1$. c. $x - k$.

	a.	b.	c.
3. $x^3 + 2x^2 + 8$	296	9	$k^3 + 2k^2 + 8$
4. $x^5 - 3x^4 + 4x$	3912	-8	$k^5 - 3k^4 + 4k$
5. $4x^4 + 3x^2 + 2$	5294	9	$4k^4 + 3k^2 + 2$

6. Check your answer to Question 1 by using the Remainder Theorem and explain what you did.
$p(-4) = -145$; This checks the answer because the Remainder Theorem says that when $p(x)$ is divided by $x - c$, the remainder is $p(c)$.

7. If the remainder when $g(x)$ is divided by $x + a$ is 4, what is the value of $g(-a)$? Justify your answer.
By the Remainder Theorem, $g(-a) = 4$.

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4-4 Lesson Master

Questions on SPUR Objectives
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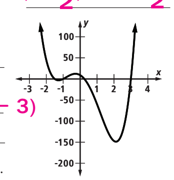
SKILLS Objective D

1. Given that $m - 2$ is a factor of $f(m) = m^3 + 6m^2 - 9m - 14$, find all the zeros of $f(m)$. 2, −7, and −1

2. Given that $y + 6$ and $y - 1$ are factors of $h(y) = 4y^4 + 24y^3 - 7y^2 - 39y + 18$, find all the zeros of h. $-6, 1, -\frac{3}{2}$, and $\frac{1}{2}$

3. At the right is the graph of $q(x) = 8x^4 - 6x^3 - 47x^2 - 24x + 9$.



a. Use the graph to find two zeros of q. −1 and 3

b. Use the Factor Theorem and your answer to Part a to find two factors of q. $(x + 1)$ and $(x - 3)$

c. Find the other two zeros of q. $\frac{1}{4}$ and $-\frac{3}{2}$

4. a. Use a CAS to write $p(z) = 14z^3 - 69z^2 + 61z + 60$ in factored form.
$p(z) = (7z + 4)(2z - 5)(z - 3)$

b. Use your answer to Part a to find the three real solutions to the equation $0 = 14z^3 - 69z^2 + 61z + 60$.
$z = -\frac{4}{7}$, $z = \frac{5}{2}$, or $z = 3$

PROPERTIES Objective H

5. Let $p(x) = (x - a) \cdot q(x)$, and let b be a zero of q.

a. Name a factor of $q(x)$. $x - b$

b. Name two zeros of p. a and b

6. *True or False.* Justify your answer.

a. A polynomial of degree n can have $n + 1$ real zeros. false; By the Number of Zeros of a Polynomial Theorem, a polynomial of degree n can have at most n zeros.

b. If $n \geq 2$, polynomial of degree n can have exactly $n - 2$ real zeros. true; By the Number of Zeros of a Polynomial Theorem, a polynomial of degree n can have at most n zeros, but can have fewer.

7. If $(t - 6)^3$ is a factor of $q(t)$, what can you say about the multiplicity of the zero 6 of $q(t)$?
The multiplicity of the zero 6 is at least 3.