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4-5 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 275–277 for objectives.

SKILLS Objectives C and E

In 1–3, give the prime factorization of the polynomial over the reals.

1. $x^2 + 3x - 4$ $(x - 1)(x + 4)$
2. $(w^2 - 9)(3w^3 + 5w^2)$ $(w + 3)(w - 3)(3w + 5)w^2$
3. $m^5 + m^3 - 12m$ $m(m^2 + 4)(m + \sqrt{3})(m - \sqrt{3})$
4. Let $p(y) = y^5 - 8y^4 + 23y^3 - 40y^2 + 60y - 48$. Given that $y^2 + 3$ is a factor of $p(y)$, 4 is a zero of $p(y)$, and -2 is a zero of $p(y)$ of multiplicity 2, find the prime factorization of $p(y)$.
$(y^2 + 3)(y - 4)(y - 2)^2$
5. What is the largest prime factor you might have to check to see if 1223 is prime? 31

In 6 and 7, determine the standard prime factorization of the number or state that it is prime.

6. 680 $2^3 \cdot 5 \cdot 17$
7. $2.43 \cdot 10^6$ $2^4 \cdot 3^5 \cdot 5^4$
8. Use the Fundamental Theorem of Arithmetic to explain why 13 is not a factor of 42,500.
Answers vary. Sample: A prime factorization of 42,500 is $2^2 \cdot 5^4 \cdot 17$. The Fundamental Theorem of Arithmetic states that every integer that is not prime has a unique prime factorization except for the order of the factors. Since 13 is prime but is not in this prime factorization of 42,500, it is not in any prime factorization of this number and therefore not a factor.

PROPERTIES Objective I

9. Prove by contradiction: There is no greatest multiple of 7. Answers vary. Sample:
Proof: Assume there is a greatest multiple of 7, and call it n. Since n is a multiple of 7, $n = 7q$ for some integer q. Now consider $n + 7$. Note that $n + 7 = 7q + 7 = 7(q + 1)$. Since $q + 1$ is an integer, $n + 7$ is a multiple of 7. But $n + 7 > n$, so this contradicts our assumption. Therefore there is no greatest multiple of 7.

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4-6 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 275–277 for objectives.

SKILLS Objective B

1. Name two integers, one positive and one negative, that are congruent to 17 modulo 5.
Answers vary. Sample: 22 and –8

In 2–5, name the smallest positive integer that makes the congruence true.

2. $w \equiv 184 \pmod 7$ $w = 2$
3. $x \equiv -123 \pmod{11}$ $x = 9$
4. $y \equiv 25{,}244 \pmod 9$ $y = 9$
5. $z \equiv 1601 \pmod{201}$ $z = 194$
6. Determine into which set, R_0, R_1, R_2, R_3, R_4, or R_5 modulo 6, the given integer falls.
a. 794 R_2
b. 1723 R_1
c. –409 R_5
d. 10,044 R_0

PROPERTIES Objective G

7. Suppose $q - r$ is a multiple of n. What can you conclude about q and r?
$q \equiv r \pmod n$
8. If $a \equiv b \pmod m$ and $17 \equiv 23 \pmod m$, what congruence statement can you make about $17a$?
$17a \equiv 23b \pmod m$

In 9–11, rewrite in the language of congruences.

9. p is 3 more than a multiple of 7. $p \equiv 3 \pmod 7$
10. $t = 45° + 90k$ where k is an integer. $t \equiv 45° \pmod{90}$

USES Objective K

11. Some companies require that you be an employee for 90 calendar days before receiving benefits. If Judy starts working for such a company on a Monday, what day of the week will it be when she becomes eligible for benefits?
Saturday

In 12 and 13, find the check digit X_c at the end of the given ISBN-10 number.

12. one edition of J. D. Salinger's *Catcher in the Rye*: 0-316-76948-X_c 7
13. one edition of Charles Dickens' *Great Expectations*: 0-312-08082-X_c 4

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4-7 Lesson Master

Questions on SPUR Objectives
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REPRESENTATIONS Objective L

In 1–3, a number in a base other than 10 is given.
a. Write the number in polynomial form.
b. Evaluate the polynomial to find the base-10 representation of the number.

1. 3021_4
a. $3 \cdot 4^3 + 0 \cdot 4^2 + 2 \cdot 4 + 1$
b. 201

2. 10101_2
a. $1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2 + 1$
b. 21

3. $4A8_{16}$
a. $4 \cdot 16^2 + 10 \cdot 16 + 8$
b. 1192

In 4–6, write the given base-10 number in a. base-2. b. base-16.

4. 17
a. 10001_2
b. 11_{16}

5. 26
a. 11010_2
b. $1A_{16}$

6. 101
a. 1100101_2
b. 65_{16}

In 7–9, perform the addition in base 2.

7. $1\,0\,0\,1_2 + 1\,1\,0\,0_2$ = 10101_2
8. $1\,1\,0\,0\,1_2 + 1\,0\,0\,0\,1_2$ = 101010_2
9. $1\,1\,1\,0\,1\,0_2 + 1\,1\,0\,0\,0_2$ = 1010010_2

10. Check your answer to Question 9 by finding the base-10 representations of the numbers.
$111010_2 = 58_{10}$ and $11000_2 = 24_{10}$; $58 + 24 = 82$; $82_{10} = 1010010_2$
11. Without converting to base 10, tell whether the binary number 101011_2 is even or odd, and explain how you know. odd; The binary number ends in 1, so the last term of the polynomial representation will be $1 \cdot 2^0 = 1$. The other terms are powers of 2 and even; the sum of 1 and any even number is odd.
12. a. What digits can be used in a base-5 representation of a number? 0, 1, 2, 3, and 4
b. What digits can be used in a base-12 representation of a number? 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B
13. a. How many digits are in the binary representation of 100,101? 17
b. How many digits are in the base-10 representation of 100101_2? 2
14. Give the base-6 representation of the base-10 number 1983. 13103_6

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5-1 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 339–343 for objectives.

PROPERTIES Objectives E and F

1. Prove that $\sqrt{23}$ is irrational.
Suppose that $\sqrt{23}$ is rational. Then $\sqrt{23} = \frac{a}{b}$ for some integers a and b with no common factors. Squaring both sides of this equation gives $23 = \frac{a^2}{b^2}$, and multiplying both sides by b^2 gives $23b^2 = a^2$. Thus a^2 is divisible by 23, and so a must also be divisible by 23. Then we can substitute $23k$ for a, where k is an integer, to get $23b^2 = (23k)^2$, or $b^2 = 23k^2$. Thereore b^2 and b are each divisible by 23, and a and b have a common factor of 23. This is a contradiction, so $\sqrt{23}$ is irrational.
2. Use proof by contradiction to show that the reciprocal of any irrational number is irrational.
Let p be an irrational number, and suppose the reciprocal q of p is rational. Then $q = \frac{a}{b}$ for some integers a and b. But $p = \frac{1}{q} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$. Then p is a quotient of integers, so p is a rational number. This is a contradiction, so the reciprocal of an irrational number must be irrational.
3. *True or False*. The difference of any two irrational numbers is irrational. If true, prove it. If false, provide a counterexample.
False; For a counterexample, let $r = s = \sqrt{2}$. Then $r - s = 0$, which is irrational.

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