Back to Lesson 3-1

Name

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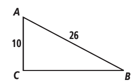
In 11–14, tell whether there is a value of θ that meets the given conditions. If so, find the value of θ. If not, explain why not.

11. $\sin\theta = 0.9$ and $\cos\theta = 0.4$ no; $\sin^2\theta + \cos^2\theta \neq 1$
12. $\csc\theta = 2$ and $\frac{\pi}{2} < \theta < \pi$ yes; $\theta = \frac{5\pi}{6}$
13. $\cos\theta = -1.5$ yes; $\forall\ \theta, -1 \le \cos\theta \le 1$
14. $\tan\theta = -1$ and $\sec\theta$ is positive yes; Answers vary. Sample $\theta = -45°$

15. Give a value of x for which each value is undefined, or say that such a value does not exist.
   a. $\sin x$ does not exist
   b. $\cos x$ does not exist
   c. $\tan x$ Answers vary. Sample: 90°
   d. $\csc x$ Answers vary. Sample: π
   e. $\sec x$ Answers vary. Sample: $\frac{3\pi}{2}$
   f. $\cot x$ Answers vary. Sample: 180°

16. Find the given values for the triangle at the right.
   a. $\sin B$ $\frac{10}{26} = \frac{5}{13}$
   b. $\cos B$ $\frac{24}{26} = \frac{12}{13}$
   c. $\tan B$ $\frac{10}{24} = \frac{5}{12}$
   d. $\sec B$ $\frac{26}{10} = \frac{13}{5}$
   e. $\csc B$ $\frac{26}{24} = \frac{13}{12}$
   f. $\cot B$ $\frac{24}{10} = \frac{12}{5}$



Name

2-9 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 142–145 for objectives.

SKILLS Objective B 1–4. Answers vary. Samples:

In 1 and 2, identify four points (x, y) on the graph of the equation. Include at least one negative value of x and at least one value of x greater than 2π.

1. $y = \sin x$ $(0, 0), \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), \left(-\frac{2\pi}{3}, \frac{-\sqrt{3}}{2}\right), \left(\frac{13\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
2. $y = \sec x$ $(0, 1), \left(\frac{5\pi}{6}, -\frac{2\sqrt{3}}{3}\right), \left(-\frac{3\pi}{4}, -\sqrt{2}\right), (3\pi, -1)$

In 3 and 4, name at least three values of θ that satisfy the given equation. Express your answers in radians.

3. $\tan\theta = \sqrt{3}$ $\frac{\pi}{3}, -\frac{5\pi}{3}, \frac{7\pi}{3}$
4. $\csc\theta = -2$ $\frac{7\pi}{6}, -\frac{5\pi}{6}, -\frac{13\pi}{6}$

In 5 and 6, a false statement is given. a. Provide a counterexample to show that the statement is false. b. Change the statement so that it is true.

5. $\sin x = \sin(x + \pi k)$, for all integers k and real numbers x.
   a. Let $x = \frac{\pi}{2}$ and $k = 1$. Then $\sin x = \sin\left(\frac{\pi}{2}\right) = 1$ but $\sin(x + \pi k) = \sin\left(\frac{3\pi}{2}\right) = -1$.
   b. $\sin x = \sin(x + 2\pi k)$ or all integers k and real numbers x.

6. $\tan x = \tan(x + \pi k)$, for all real numbers k and real numbers x.
   a. Let $x = 0$ and $k = 0.1$. Then $\tan x = \tan 0 = 0$ but $\tan(x + \pi k) = \tan 0.1 \approx 0.1$.
   b. $\tan x = \tan(x + \pi k)$ or all integers k and real numbers x.

PROPERTIES Objective E

7. Describe the end behavior of the cosecant function. The values of the cosecant function neither approach a finite value nor increase or decrease without bound as the absolute value of x increases.
8. What is the value of $\lim_{x\to\infty} \cos x$? Explain your answer. no; The limit does not exist because the values of cos x oscillate between –1 and 1 as x increases.
9. a. Give equations of any horizontal asymptotes of the graph of $y = \sin x$, or say that they don't exist. does not exist
   b. Give equations of any horizontal asymptotes of the graph of $y = \frac{\sin x}{x^2}$, or say that they don't exist. $y = 0$

Name

3-1 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 216–219 for objectives.

SKILLS Objective B

In 1–4, let r and s be defined by $r(x) = 4 \cdot 2^x$ and $s(x) = 3 \cdot 4^x$. Find an equation for

1. $r + s$. $(r + s)(x) = 4 \cdot 2^x + 3 \cdot 4^x$
2. $r - s$. $(r - s)(x) = 4 \cdot 2^x - 3 \cdot 4^x$
3. $r \cdot s$. $(r \cdot s)(x) = 12 \cdot 2^x \cdot 4^x = 12^{3x}$
4. $\frac{r}{s}$. $\left(\frac{r}{s}\right)(x) = \frac{4 \cdot 2^x}{3 \cdot 4^x} = \frac{4}{3} \cdot 2^{-x}$
5. If $f(x) = \frac{x + 2}{x - 1}$ and $g(x) = \sqrt{2x - 1}$, what is the domain of $f \cdot g$? $\left[\frac{1}{2}, 1\right) \cup (1, \infty)$
6. Let p and q be sequences defined for $n \ge 1$ by $p_n = 3n^3 - 3n$ and $q_n = n + 1$. Find an explicit formula for $\left(\frac{q}{p}\right)_n$. $\left(\frac{q}{p}\right)_n = \frac{1}{3n^2 - 3n}\ \forall\ n \in \mathbb{Z}, n > 1$

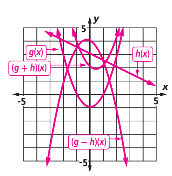
USES Objective J

In 7 and 8, use this information. Suki wants to take a bath. When she turns the faucet all the way on, the water level in her tub rises at a rate of 1.75 inches per minute. However, she has a leaky plug that causes the water level to fall at a rate of 0.2 inches per minute.

7. a. Find formulas for $r(t)$, the number of inches the faucet has caused the water level to rise after t minutes, and $f(t)$, the number of inches the leaky plug has caused the water level to fall after t minutes. $r(t) = 1.75t$; $f(t) = -0.2t$
   b. Use your answer to Part a to find a formula for $h(t)$, the height of the water after t minutes. $h(t) = 1.55t$
8. a. How long will it take for the water level to reach 10 inches? about 6 minutes and 27 seconds
   b. After she turns off the water, how long will Suki have to soak before the water level falls to 5 inches? 25 minutes

REPRESENTATIONS Objective L

9. On the axes at the right, sketch a graph of each function over the domain $-5 \le x \le 5$. Be sure to label each curve.
   a. the function g with equation $g(x) = x^2 - 1$
   b. the function h with equation $h(x) = -\frac{1}{2}x + 3$
   c. $g + h$
   d. $h - g$



Name

3-2 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 216–219 for objectives.

SKILLS Objectives B and C

In 1–4, formulas for real functions p and q are given. Find an equation for and the domain of

a. $p \circ q$. b. $q \circ p$.

1. $p(x) = x^2$, $q(x) = \log x$
   a. $p \circ q(x) = (\log x)^2$; $(0, \infty)$
   b. $q \circ p(x) = \log(x^2)$; $(-\infty, 0) \cup (0, \infty)$
2. $p(x) = |x|$, $q(x) = \sqrt{-x}$
   a. $p \circ q(x) = |\sqrt{-x}|$; $(-\infty, 0]$
   b. $q \circ p(x) = \sqrt{-|x|}$; The function is undefined since there are no values in its domain.
3. $p(x) = \frac{(x + 2)(3x - 1)}{x - 3}$, $q(x) = \frac{1}{x^2}$
   a. $p \circ q(x) = \frac{\left(\frac{1}{x^2} + 2\right)\left(-\frac{3}{x^2} - 1\right)}{\frac{1}{x^2} - 3}$; $(-\infty, 0) \cup \left(0, \frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$
   b. $q \circ p(x) = \left(\frac{x - 3}{(x + 2)(3x - 1)}\right)^2$; $(-\infty, -2) \cup \left(-2, \frac{1}{3}\right) \cup \left(\frac{1}{3}, 3\right) \cup (3, \infty)$
4. $p(x) = x^3 - 2x + 4$, $q(x) = 2e^x$
   a. $p \circ q(x) = 8e^{3x} - 4e^x + 4$; $(-\infty, \infty)$
   b. $q \circ p(x) = 8e^{x^3 - 2x + 4}$; $(-\infty, \infty)$
5. Let $f(x) = e^x$ and $g(x) = \ln x$.
   a. Find an equation for $f \circ g$ and give its domain. $f \circ g(x) = x$; $(0, \infty)$
   b. Find the domain of $h : x \to x$. $(-\infty, \infty)$
   c. Explain why your answers to Parts a and b are not the same. The domain of h is $\mathbb{R}$, but g is not defined when $x \le 0$.

In 6–9, an equation for the composite of two functions m and n is given. Identify possible equations for m and n. Answers vary. Samples:

6. $m \circ n(x) = e^{2x+1}$ $m(x) = e^x$ and $n(x) = 2x + 1$
7. $m \circ n(t) = \sin^2 t + 4$ $m(t) = t^2 + 4$ and $n(t) = \sin t$
8. $m \circ n(x) = \sqrt{\log_2 x}$ $m(x) = \sqrt{x}$ and $n(x) = \log_2 x$
9. $m \circ n(x) = (2x - 12)^5$ $m(x) = x^5$ and $n(x) = 2x - 12$
10. Let $f(x) = |\cos(4^x)|$. Write f as the composite of Answers vary. Samples:
   a. two simpler functions. $g \circ h$, where $g(x) = |x|$ and $h(x) = \cos(4^x)$
   b. three simpler functions. $g \circ h \circ k$, where $g(x) = |x|$, $h(x) = \cos(x)$, and $k(x) = 4^x$